Long distance dispersal events accelerate range expansions

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Everything Disperses to Miami,
Non-local dispersal in ecology and epidemiology, 14/12/12

Range expansion phenomenon

Growing number of observations of range expansions mainly because of

- climate changes (climatic niches shifting);
- biological invasions;
- human activities (transportation of species).

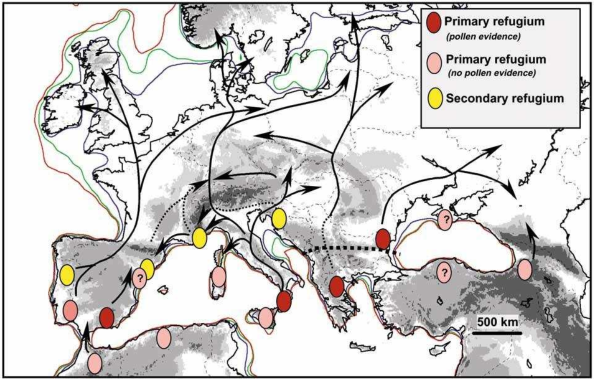
Range expansion is the result of

- **Dispersal**
  - Local diffusion: movement into adjacent habitat;
  - Non-local dispersion: long distance dispersal.
- **Population growth**
  - Logistic growth: competition (for food and space) leads to negative density dependence.
  - Allee effect: lower fertility at low density (*Allee 1932*).
    Examples: mate limitation, consanguinity, cooperative defense or feeding,...

→ Important issue: the speed of range expansion.

Reid's paradox of rapid plant migration

(*Reid, 1899*): Recolonization from Southern refugia at the end of the last glacial period ($\sim 10\,000$ years ago).
Current distribution of oak in Europe cannot be explained by diffusive dispersal.



(*Petit et al., 2002*)

$\rightarrow$ recolonization was faster than expected

Fast propagation in reaction-dispersion equations

- Existence of **cryptic refugia** accelerate propagation. (*Mc Lachlan et al. 2005, Provan and Bennett 2008)*

(*Hamel and Roques 2010, Roques et al. 2011*) : solutions of RD equations with *EU initial data* accelerate.

- **Long distance dispersal** events increase the dispersal capability. (*Skellam 1951, Clark et al., 1998*)

Models: Integro-differential equations:

$$\frac{\partial u}{\partial t}(t,x) = \int_{-\infty}^{+\infty} J(|x-y|)\big(u(t,y)-u(t,x)\big)\,dy + f\big(u(t,x)\big)$$

Numerical observations and formal computations: Infinite asymptotic spreading speed and accelerating rate of spread if the dispersal kernel J is "fat-tailed" (*Mollison 1977, Kot et al. 1996, Medlock and Kot 2003*).

$\rightarrow$ **How does the dispersal mode impact the spreading speed?**

Integro-differential model, basic assumptions

$$\frac{\partial u}{\partial t}(t,x) = \int_{\mathbb{R}} J(x-y)\big(u(t,y)-u(t,x)\big)\,dy \quad + \quad f\big(u(t,x)\big)$$

Dispersal term — Growth term

Initial condition u_0 :

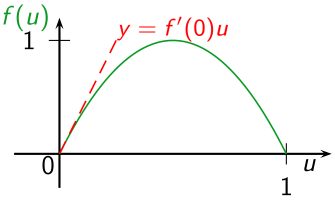
$u_0 : \mathbb{R} \to [0,1]$ is $\mathcal{C}^0$ function, compactly supported and $u_0 \not\equiv 0$.

Monostable term f:

$$f(0) = f(1) = 0,\; f(s) > 0 \text{ for all } s \in (0,1),\; \text{and } f'(0) > 0.$$

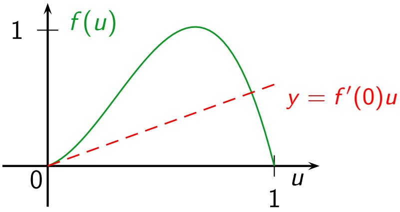
Logistic growth – KPP case

$0 < f(u) \le f'(0)u$



Weak Allee case

$f(u)/u$ not maximal at 0.



Integro-differential model, basic assumptions

$$\frac{\partial u}{\partial t}(t,x) = \underbrace{\int_{\mathbb{R}} J(x-y)\big(u(t,y)-u(t,x)\big)dy}_{\text{Dispersal term}} + \underbrace{f\big(u(t,x)\big)}_{\text{Growth term}}$$

Initial condition u_0 :

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Dispersal kernel J: $J(x-y)$ the probability distribution of jumping from location y to location x.

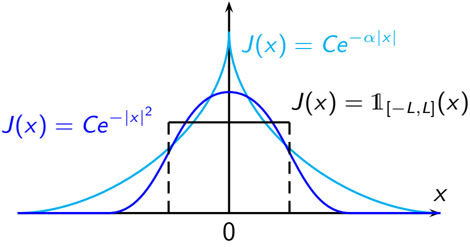
$$J \in \mathcal{C}^0,\ J \geq 0,\ J(x) = J(-x),\ \int_{\mathbb{R}} J = 1 \text{ and } \int_{\mathbb{R}} |x| J(x)dx < \infty.$$

Dispersal kernel assumptions

Exponentially bounded kernel (EB)
(Diekmann 1979, Thieme 1979, Schumacher 1980, Weinberger 1982, Coville et al. 2008)

Definition
$\exists\, \alpha > 0$ s.t. J satisfies

$$0 \leq J(x) \leq e^{-\alpha|x|}, \text{ for large } x.$$



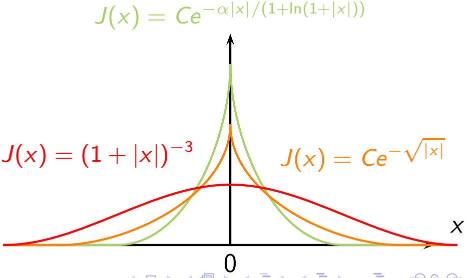
Exponentially unbounded kernel (EU)
(Medlock and Kot 2003, Yagisita 2009)

Definition
J satisfies

$$J'(x)/J(x) \to 0 \text{ as } |x| \to +\infty.$$

$$\Rightarrow\ J(x)\, e^{\alpha|x|} \to +\infty \text{ as } |x| \to +\infty$$

for all $\alpha > 0$.

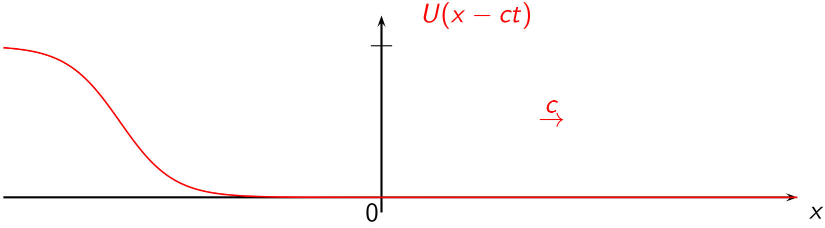


Exponentially bounded kernel: classical results

(*Schumacher, 1980; Coville et Dupaigne, 2007*): *existence of traveling wave solutions* $u(t,x) = U(x-ct)$ *for speed* $c \geq c^* > 0$;

$$\int_{\mathbb{R}} J(x-y)(U(y)-U(x))dy + cU'(y) + f(U(y)) = 0, \quad \text{in } \mathbb{R}$$

$$U(-\infty) = 1 \quad \text{and} \quad U(+\infty) = 0 \quad \text{and} \quad U' < 0 \text{ in } \mathbb{R}.$$



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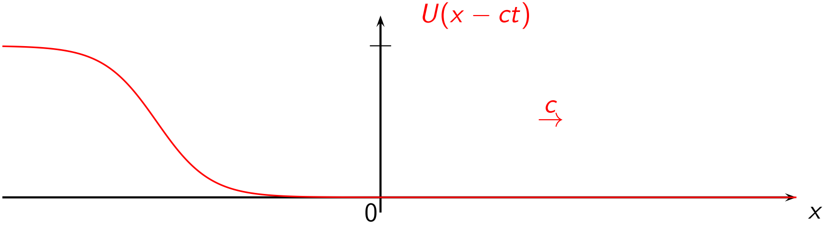

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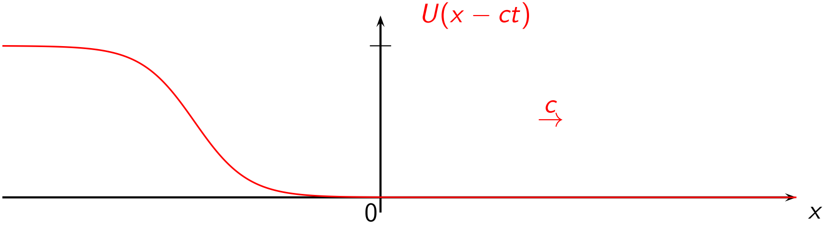

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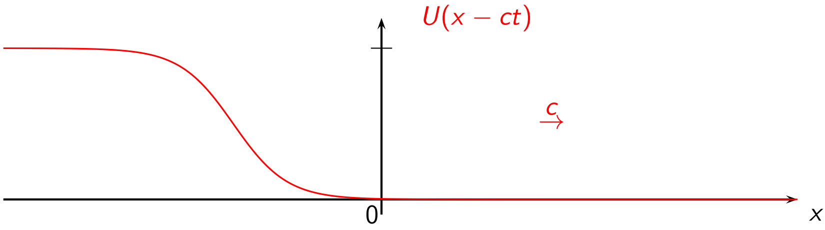

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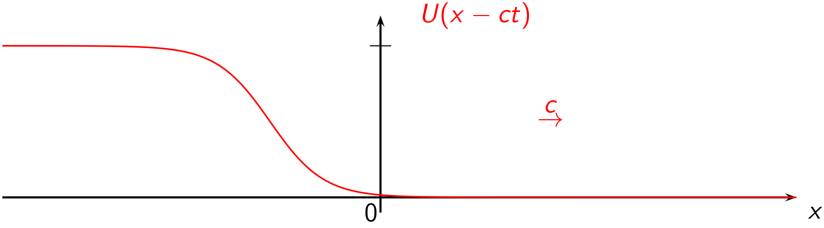

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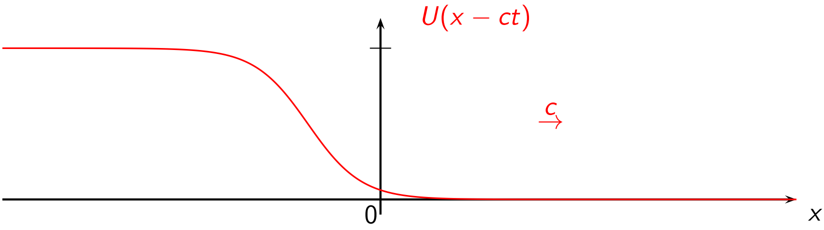

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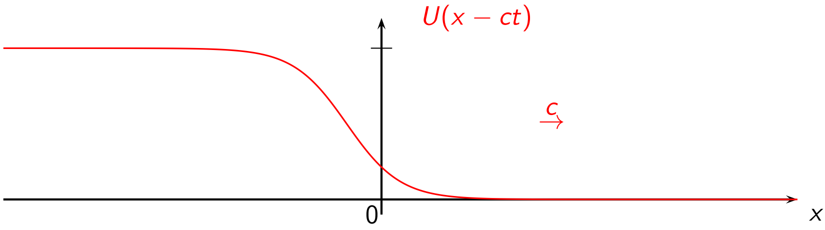

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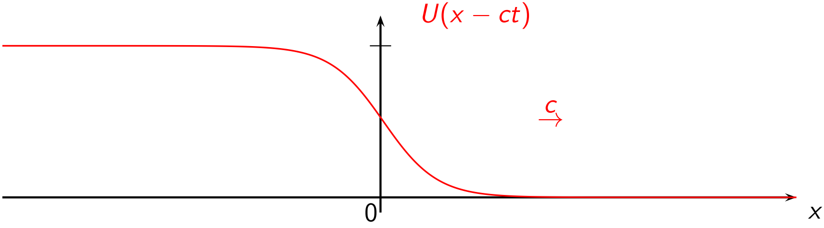

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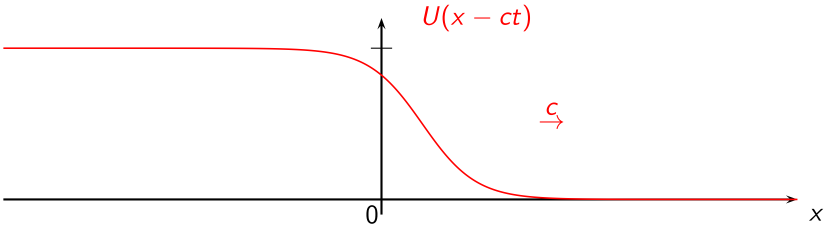

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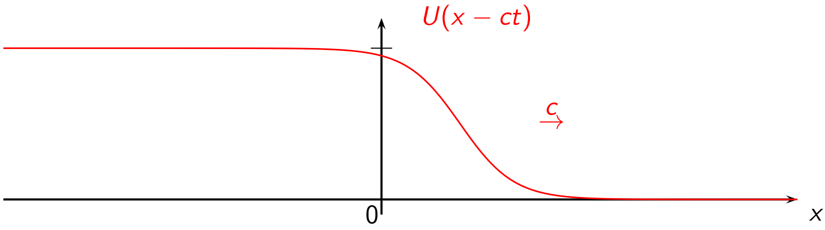

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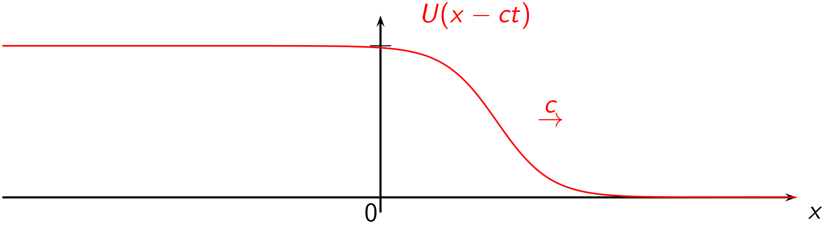

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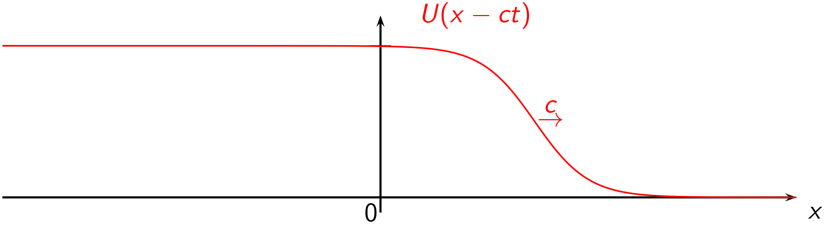

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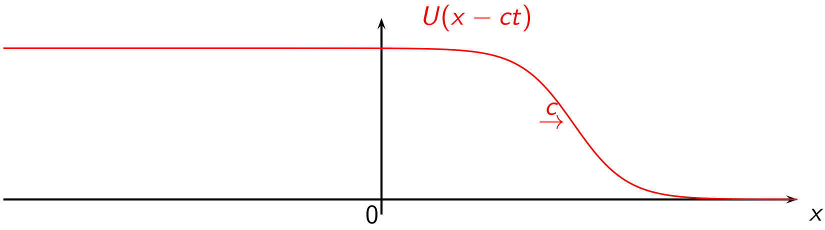

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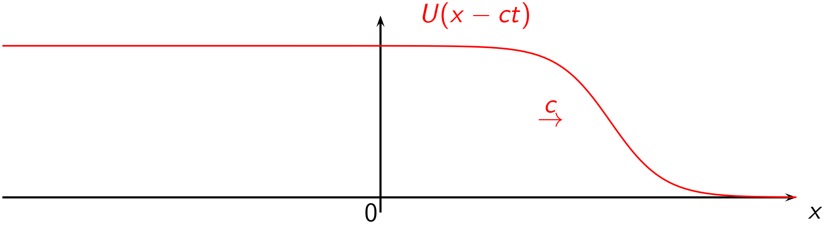

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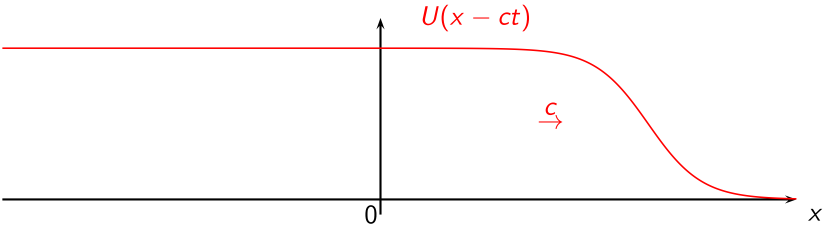


Exponentially bounded kernel: classical results

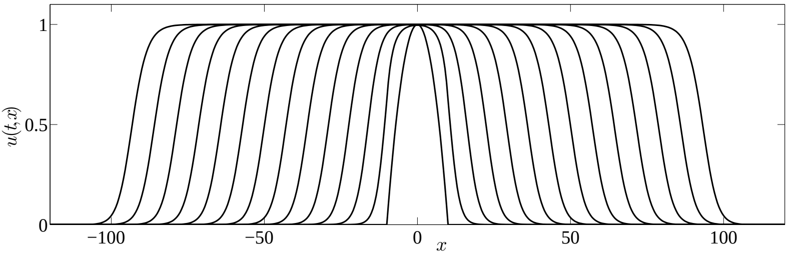
The spreading speed c of a solution u of IDE is defined by:

$$\limsup_{t\to\infty} u(t,|x|-wt) = 0 \text{ if } w > c$$
$$\liminf_{t\to\infty} u(t,|x|-wt) = 1 \text{ if } 0 < w < c$$

(*Lutcsher et al., 2005*): *if u_0 is compactly supported, the spreading speed c of u satisfies $c = c^*$, the minimal speed of traveling fronts.*

→ spreading speed remains finite.

Numerical obs: the solution converges to a traveling front with constant profile.



EU kernel: infinite spreading speed

Hypothesis: Exponentially unbounded kernel

$J(x)$ is decreasing for all $x \geq 0$, J is a $\mathcal{C}^1$ function for large x and

$$\frac{J'(x)}{J(x)} \to 0 \text{ as } |x| \to +\infty.$$

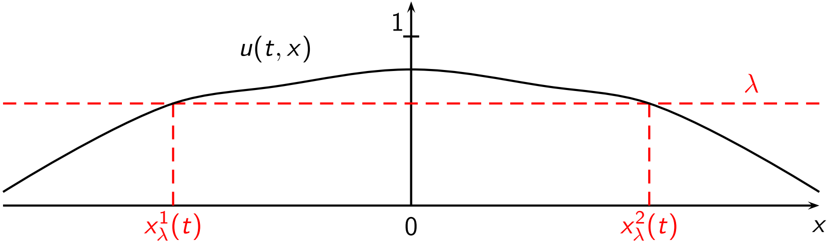
Theorem 1 *(Garnier, 2011)*

If the kernel J is EU, the asymptotic spreading speed of $u(t,x)$ is infinite.

EU kernel: Level sets $E_\lambda(t)$

For all $\lambda \in (0,1)$, we define the level set $E_\lambda(t)$ by:

$$E_\lambda(t) := \{x \in \mathbb{R},\ u(t,x) = \lambda\},$$



From Theorem 1, for all $\lambda \in (0,1)$,

$$\lim_{t\to+\infty} \frac{|x_\lambda^1(t)|}{t} = \lim_{t\to+\infty} \frac{|x_\lambda^2(t)|}{t} = +\infty.$$

EU kernel: Lower and Upper bounds of $E_\lambda(t)$

We get bounds for the position of the level sets, which explicitly depend on the dispersal kernel J and the reaction term f.

Theorem 2 (Garnier, 2011)

Let J be EU. Then there exists $\rho \geq f'(0)$ such that for any $\lambda \in (0,1)$, and $\varepsilon > 0$, every element $x_\lambda(t) \in E_\lambda(t)$ verifies:

$$J^{-1}\left(e^{-(f'(0)-\varepsilon)t}\right) \leq |x_\lambda(t)| \leq J^{-1}\left(e^{-\rho t}\right) \quad \text{for large } t.$$

Two additional hypotheses for the upper bound:

Hyp. 1		**Hyp. 2**
There exists $\nu_0 \in (0,1)$ such that $$\int_{\mathbb{R}} J(z)^{\nu_0} dz < \infty.$$	or	There exists $C > 0$, such that $$\left|\frac{J'(x)}{J(x)}\right| \sim C\frac{1}{|x|} \text{ as } |x| \to \infty.$$

EU kernel: some examples

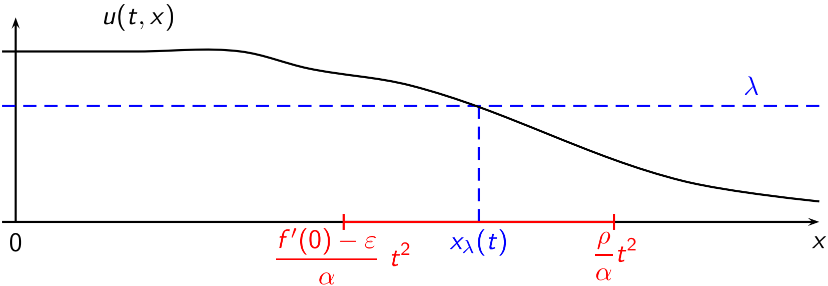
Kernel J satisfying Hyp. 1 but not Hyp. 2:

- J is logarithmically power-like and sub-linear as $|x| \to \infty$,

$$J(x) = Ce^{-\alpha\sqrt{|x|}} \text{ for large } |x|, \; \alpha > 0, \; C > 0.$$

$\to$ every $x_\lambda(t) \in E_\lambda(t)$ satisfies for any $\varepsilon > 0$

$$\frac{(f'(0) - \varepsilon)}{\alpha} t^2 \le |x_\lambda(t)| \le \frac{\rho}{\alpha} t^2 \text{ for large } t;$$



EU kernel: some examples

Kernel J satisfying Hyp. 2:

- J decays algebraically as $|x| \to \infty$,

$$J(x) = C|x|^{-\alpha} \text{ for large } |x|, \ \alpha > 2, \ C > 0,$$

$\to$ every $x_\lambda(t)$ propagates exponentially fast as $t \to +\infty$, for any $\varepsilon > 0$

$$e^{\frac{f'(0)-\varepsilon}{\alpha} t} \ \leq \ |x_\lambda(t)| \ \leq \ e^{\frac{\tilde{\rho}}{\alpha} t} \ \text{ for large } t,$$

(*Cabré and Roquejoffre 2009*): Similar results with fractional Laplacian diffusion:

$$\partial_t u(t,x) = \int_{\mathbb{R}} \frac{c_\alpha}{|x-y|^{1+2\alpha}} \big(u(t,y) - u(t,x)\big) dy + f(u(t,x))$$

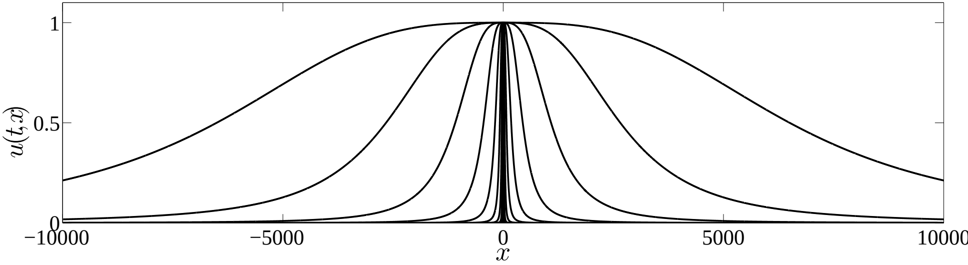
Let $\alpha \in (0,1)$ *and set* $c^* = f'(0)/(1+2\alpha)$, *then*

$$\lim_{t\to+\infty} \inf_{|x|\leq e^{ct}} u(t,x) = 1 \text{ if } c < c^* \ \Big| \ \lim_{t\to+\infty} \sup_{|x|\geq e^{ct}} u(t,x) = 0 \text{ if } c > c^*.$$

EU kernel: qualitative results

Global behavior of the solution $u(t, x)$:

- The rate of spread increases in time like $J^{-1}(e^{-\gamma t})/t$.
  $\rightarrow$ acceleration of the propagation and infinite asymptotic spreading speed,
- The profile of the front tends to flatten with time.
  $\rightarrow$ no convergence to traveling wave solution (*Yagisita, 2009*);
  $\rightarrow$ the leading edge of the population spreads faster.



Conclusions

Real dichotomy

Exponentially Unbounded kernels:

- Infinite asymptotic spreading speed;
- The positions of the level sets accelerate with time faster than $J^{-1}(e^{-\gamma t})$;
- EU kernels = fat tailed kernels.

Exponentially Bounded kernels and RD equations:

- Finite spreading speed;
- The solution converges to a traveling front with constant profile;
- EB kernels = thin tailed kernels.

Taking Long Distance Dispersal events into account is of critical importance.

Conclusions

Thank you for your attention

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