The ideal free strategy with weak Allee effect

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Quantify movement phenotype via flux: diffusive and advective

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- $m(x) > 0$ is nonconstant (spatially inhomogeneous)
- Semi-trivial steady states: $(u^*, 0)$ and $(0, v^*)$
- Is there a strategy $P(x)$ which cannot be invaded?

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- We call $P = \ln m$ an Ideal Free Strategy (IFS).

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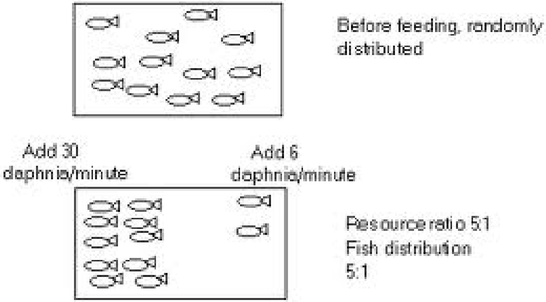

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- Biologically, $P = \ln m$ is a global ESS.
- Main Question: Does this result still hold when $u(m - u - v)$ is replaced by $u^2(m - u - v)$ in model (1)?

Modified Model

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- u is subject to weak Allee effect (species no longer have the same population dynamics)
- Interplay between IFS and weak Allee effect
- Invasion dynamics not useful for any $\beta \in [0, \infty)$

$\beta = 0$ case

Theorem 2

Suppose $m \in C^2(\bar{\Omega})$ is positive and non-constant. Then for $\beta = 0$ and any μ, $\nu > 0$, any solution (u, v) of (2) with nonnegative, not identically zero initial data converges to $(m, 0)$ in $L^\infty(\Omega)$ as $t \to \infty$.

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- IFS offsets the weak Allee effect

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  $$\frac{dE}{dt} = -\mu \int_\Omega \frac{2m|\nabla(u/m)|^2(1-(u/m))}{(u/m)^3} - \nu \int_\Omega |\nabla v|^2 - \int_\Omega ((m-u)^2 - v^2)(m-u-v).$$

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So $\frac{d}{dt}\int_\Omega \frac{m^2}{u} \leq -\eta/2 < 0$ for all $t > 0$. Therefore
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- Monotone dynamical system theory to conclude $(m, 0)$ is globally asymptotically stable.

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Theorem 3

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- Again, u is sole winner as IFS is able to still offset the Allee effect.
- Proof for Theorem 3 is similar to proof of Theorem 2 but more technical. Eliminating the possibility of positive coexistence states is most difficult part.

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- For the $\beta >> 1$ case, we can show that $(0, v^*)$ is unstable.

  We conjecture that u (IFS) should be the sole winner as in Theorem 2.

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Fundamentally different:

- Theorem 1 no longer holds, i.e. the winning strategy is no longer a "resource matching" strategy.
- Biological explanation and mathematical justification?

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Numerical example:

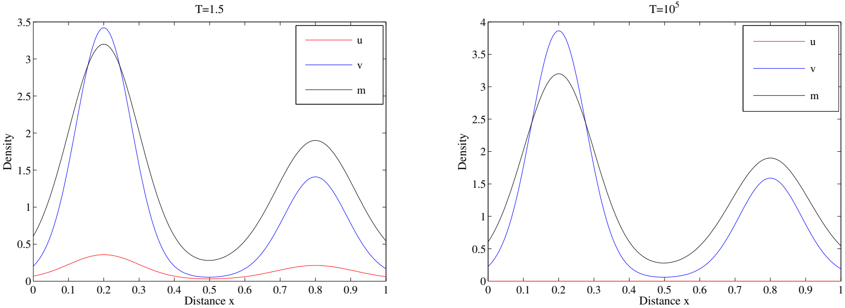

Figure: $m(x) = 3e^{-50(x-.2)^2} + 1.7e^{-40(x-.8)^2} + .2$ (black) and u (red) and v (blue), $\mu = 1000$, $\nu = 1000$, $\beta = 1.7$ a) two species at $T = 1.5$, b) $T = 10^5$.

Intermediate $\beta > 1$ case

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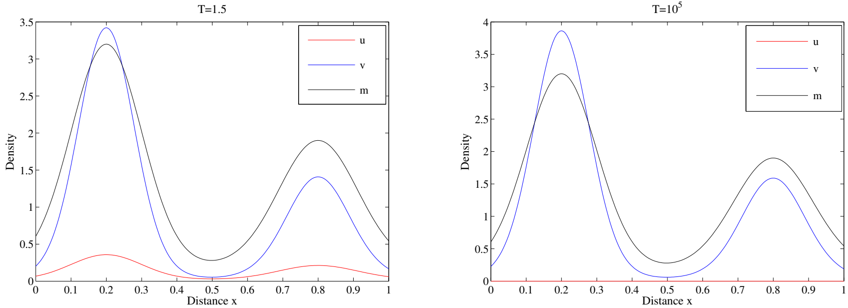

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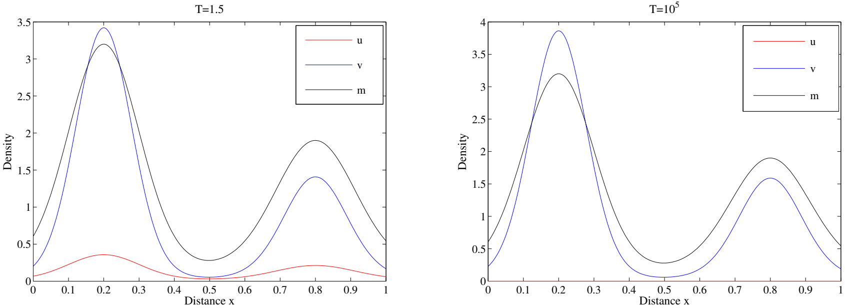

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- The growth rate for u near $x = 0.8$ is $m(x) - v(x,t) > 0$ for all $t > T_0$.
- For β in this range, v can defeat u even when u has significant initial numbers.

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- For $\beta = 1$, coexistence as both species are ideal free dispersers
- For intermediate $\beta > 1$, the ideal free strategy is no longer is optimal as it can be invaded.

References

- R.S. Cantrell, C. Cosner, and Y. Lou, Evolution of dispersal and ideal free distribution, Math Bios. Eng., Vol 7 (2010) 17-36.
- I. Averill, Y. Lou, and D. Munther, On several conjectures from evolution of dispersal, J. Biol. Dynamics, 6 (2012) 117-130.
- D. Munther, The ideal free strategy with weak Allee effect, J. Differential Equations, (2013) in press.

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