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Title: New methods in the classification of class VII surfaces.

Abstract: The classification of complex surfaces is not completed yet. The most important gap in the Kodaira-Enriques classification table concerns the Kodaira class VII, i.e. the class of surfaces X having $\mathrm{kod}(X) = 1$, $b_1(X) = 1$. These surfaces are also interesting from a differential topological point of view, because they are non-simply connected 4-manifolds with definite intersection form.

The GSS conjecture, which, if true, would complete the classification of this class, can be stated as follows:

The GSS conjecture. Any minimal class VII surface with $b_2 > 0$ is a Kato surface.

The standard approach for proving the GSS conjecture has two steps corresponding to the following two conjectures considered by experts to be more accessible:

Conjecture 1. Any minimal class VII surface with $b_2 > 0$ has a cycle of rational curves.

Conjecture 2. Any minimal class VII surface with $b_2 > 0$ containing a cycle of rational curves is a Kato surface.

My method for proving Conjecture 1 starts with the observation that the absence of a cycle of curves in a class VII surface X implies the appearance of a smooth compact connected component in a certain moduli space of polystable bundles (PU(2)-instantons) on X. For $b_2(X) \leqslant 3$ I showed that the presence of such a component leads to a contradiction.

Recently, in a joint article with G. Dloussky, we have developed a new strategy for Conjecture 2: the idea is to study algebraic deformations of the singular surface Y obtained by contracting a cycle C of r rational curves in a minimal class VII surface X. We proved that, assuming $r < b_2(X) \leqslant 11$, the singular contraction Y will be smoothable by rational surfaces. This result leads to an interesting problem relating projective algebraic geometry to non-Kählerian complex geometry: classify families $(X_z)_{z \in D^*}$ of rational surfaces which converge to a surface Y with a single singularity, which is a cusp.

References

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