Multiple steady states for a competition system supporting an ideal-free distribution

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Abstract

In this talk we will discuss existence of steady state solutions for the competitive system

$$\begin{cases} \dfrac{\partial u}{\partial t} = \nabla \cdot \left[\alpha(x) \nabla \dfrac{u}{m} \right] + u(m(x) - u - bv) \ \text{ in } \Omega, \ t > 0, \\ \dfrac{\partial u}{\partial t} = \nabla \cdot [\beta(x) \nabla v] + v(m(x) - cu - v) \ \text{ in } \Omega, \ t > 0, \\ \nabla \dfrac{u}{m} \cdot \hat{n} = \nabla v \cdot \hat{n} = 0 \ \text{ on } \partial\Omega, \ t > 0, \end{cases}$$

which supports an *ideal free distribution* for the first species, i.e. admits a positive steady state which matches the per-capita growth rate. Previous results have stated that when $b = c = 1$ the ideal free distribution is an evolutionary stable and neighborhood invader strategy, that is the species with density v always goes extinct. We will analyze how the interaction coefficients b and c influence the structure of the steady state solutions of the system. In particular, how and to what extent the advantage derived from ideal free dispersal continues when there is a trade off relative to competitive impact, for example when $b > 1$ but $c < 1$. To understand this case, we will study the interplay between the inter-specific competition coefficients b, c and the diffusion coefficients $\alpha(x)$ and $\beta(x)$ on the critical values for stability of semi-trivial steady states. We will also show that under certain regimes the system sustains multiple positive steady states.