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Title: On scalar curvature, minimal surfaces, and the isoperimetric problem in the large

Abstract: A small geodesic ball at a point of positive scalar curvature has more volume than a Euclidean ball with the same perimeter. In fact, the magnitude of the scalar curvature can be computed as an isoperimetric deficit of the geodesic ball.

This classical observation has a global counterpart that we have recently established in joint work with O. Chodosh, Y. Shi, and H. Yu: Let $(M, g) \neq \mathbb{R}^3$ be an asymptotically flat Riemannian 3-manifold with non-negative scalar curvature. For every sufficiently large amount of area, there is a unique region of largest volume whose perimeter has that area. Moreover, these large solutions of the isoperimetric problem are nested and their isoperimetric deficit from Euclidean space encodes the ADM mass of $(M, g)$. This confirms a longstanding conjecture of H. Bray, G. Huisken, and S.-T. Yau.

The goal of my lecture is to explain this effective version of the positive mass theorem, its relation with a conjecture of R. Schoen (established in joint work with O. Chodosh), and several repercussions on classical rigidity questions in geometry.