Plenary Talk Sessions

(Prado, Biltmore)

Friday Dec 14	
9:00 – 10:00	Registration
10:00 – 10:15	Conference opening
10:15 – 11:15	Richard Schoen *Quasi-local mass and comparison geometry*
11:45 – 12:45	Piotr Chruściel *Stationary black holes with negative cosmological constant*
12:45 – 14:30	Lunch break
14:30 – 15:30	Hubert Bray *On the geometry of dark matter*
15:45 – 16:45	Eric Woolgar *Conditions of curvature-dimension type in relativity and Lorentzian geometry*

Saturday Dec 15	
9:00 – 10:00	Mu-Tao Wang *Quasilocal mass at null infinity*
10:15 – 11:15	Daniel Pollack *Spacetime Topology and Singularities*
11:45 – 12:45	Lan-Hsuan Huang *Mass rigidity for hyperbolic manifolds*
12:45 – 14:30	Lunch break
14:30 – 15:30	James Isenberg *Cauchy horizons in vacuum solutions of Einstein's equations with non-closed generators*
15:45 – 16:45	Justin Corvino *On constraint deformation and the dominant energy condition*
17:30	Banquet at Fontana in Biltmore

Sunday morning, Dec 16	
9:00 – 10:00	Carla Cederbaum *On special hypersurfaces of the Schwarzschild spacetimes and related uniqueness theorems*
10:15 – 11:15	Michael Eichmair *Isoperimetric Structure of Asymptotically Flat* 3-*Manifolds: An Overview*

Short Talk Session

(Ungar 402, University of Miami)

Sunday afternoon, Dec 16	
14:00 – 14:25	Melanie Graf *Rigidity of asymptotically* $AdS_2 \times S^2$ *spacetimes*
14:30 – 14:55	Didier A. Solis Gamboa *Towards a classification of Einstein null hypersurfaces in RW-spacetimes*
15:00 – 15:25	Eleni Kontou *Quantum strong energy inequality and the Hawking singularity theorem*
15:25 – 15:45	20-minute break
15:45 – 16:10	Carlos Vega *Splitting Spacetime*
16:15 – 16:40	Armando Cabrera Pacheco *On Bartnik mass estimates of CMC Bartnik data*
16:45 – 17:10	Eric Ling *Using timelike completeness to prove inextendibility of spacetimes in low regularity*

Title and Abstract

Plenary Talks

Hubert Bray (Duke University)

Title: On the Geometry of Dark Matter

Abstract: General relativity is the idea that matter curves spacetime. However, according to observations, the universe is much more curved than would be predicted by the matter which can be seen. Even after introducing a cosmological constant (dark energy) to account for curvature on the scale of the universe, there still exists extra curvature on the scale of galaxies. This large-scale unexplained curvature, when interpreted as dark matter, actually makes up most of the mass of galaxies. In this talk, we'll discuss geometric explanations for dark matter, as well as how well their predictions agree with observations.

Carla Cederbaum (University of Tübingen)

Title: On special hypersurfaces of the Schwarzschild spacetimes and related uniqueness theorems

Abstract: The Schwarzschild spacetimes of positive mass are well known to possess a unique "photon sphere" – meaning a cylindrical, timelike hypersurface P such that any null geodesic initially tangent to P remains tangent to P – in all dimensions. We will show that they also possesses a rich family of spatially spherically symmetric "photon surfaces" – general timelike hypersurfaces P such that any null geodesic initially tangent to P remains tangent to P. This generalizes a result of Foertsch, Hasse, and Perlick from 2+1 to higher dimensions.

Furthermore, we will show that the Schwarzschild spacetimes are indeed the only static, vacuum, asymptotically flat spacetimes possessing static black hole horizons, photon spheres, and/or what Greg Galloway and I call "equipotential" photon surfaces, again in all dimensions. Physically speaking, this asserts that static, vacuum, equipotential photon surfaces have no hair.

The above results are joint work with Greg Galloway. They build on a rigidity result for asymptotically flat Riemannian manifolds of non-negative scalar curvature with special umbilic, CMC, constant scalar curvature inner boundary components that globally support a harmonic function satisfying an overdetermined system of Dirichlet and Neumann conditions on the inner boundary. The proof of this rigidity result generalizes and extends Riemannian and conformal geometry arguments

first introduced by Bunting and Masood-ul-Alam in their proof of static black hole uniqueness, a higher dimensional analog by Gibbons, Ida, and Shiromizu, and previous joint work with Greg Galloway on the uniqueness of photon spheres. It relies on Schoen and Yau's higher dimensional positive mass theorem as well as on a result by McFerron and Szekelyhidi.

We will also discuss a recent generalization of this rigidity result to sub-extremal Reissner-Nordstrm as well as a related photon sphere and static black hole horizon uniqueness result in an electro-vacuum context in arbitrary dimension by Jahns. Her result extends previous results by Bunting and Masood-ul-Alam and previous joint work of Greg Galloway and the speaker.

Justin Corvino (Lafayette College)

Title: On constraint deformation and the dominant energy condition

Abstract: We will survey some deformation results for the constraints operator, with a focus on the dominant energy condition, highlighting certain features and raising some natural questions.

Piotr Chruściel (University of Vienna)

Title: Stationary black holes with negative cosmological constant

Abstract: I will present a construction of large families of singularity-free stationary solutions of Einstein equations, for a large class of matter models including vacuum, with a negative cosmological constant. The solutions, which are of course real-valued Lorentzian metrics, are determined by a set of free data at conformal infinity, and the construction proceeds through elliptic equations for complex-valued tensor fields. One thus obtains infinite dimensional families of both strictly stationary spacetimes and black hole spacetimes.

Michael Eichmair (University of Vienna)

Title: Isoperimetric Structure of Asymptotically Flat 3-Manifolds: An Overview

Abstract: A small geodesic ball at a point of positive scalar curvature has more volume than a Euclidean ball with the same perimeter. In fact, the magnitude of the scalar curvature can be computed as an isoperimetric deficit of the geodesic ball.

This classical observation has a global counterpart that we have recently established in joint work with O. Chodosh, Y. Shi, and H. Yu: Let $(M, g) \neq \mathbb{R}^3$ be an asymptotically flat Riemannian 3-manifold with non-negative scalar curvature. For every sufficiently large amount of area, there is a unique region of largest volume whose perimeter has that area. Moreover, these large solutions of the isoperimetric problem are nested and their isoperimetric deficit from Euclidean space encodes the ADM mass of (M, g). This confirms conjectures of H. Bray, G. Huisken, and S.-T. Yau.

The goal of my lecture is to explain this *effective* version of the positive mass theorem and its relation to a question of R. Schoen (established in joint work with O. Chodosh).

Lan-Hsuan Huang (University of Connecticut)

Title: Mass rigidity for hyperbolic manifolds

Abstract: We will present a variational approach to the rigidity of positive mass theorem for asymptotically (locally) hyperbolic manifolds. The result was previously known only for spin manifolds.

James Isenberg (University of Oregon)

Title: Cauchy horizons in vacuum solutions of Einstein's equations with non-closed generators

Abstract: We consider analytic, vacuum spacetimes that admit compact, non-degenerate Cauchy horizons. Many years ago we proved that, if the null geodesic generators of such a horizon were all *closed* curves, then the enveloping spacetime would necessarily admit a non-trivial, horizon-generating Killing vector field. Using a slightly extended version of the Cauchy-Kowaleski theorem one could establish the existence of infinite dimensional, analytic families of such 'generalized Taub-NUT' spacetimes and show that, generically, they admitted *only* the single (horizon-generating) Killing field alluded to above. In this talk we relax the closure assumption and analyze vacuum spacetimes in which the generic horizon generating null geodesic densely fills a 2-torus lying in the horizon. In particular we show that, aside from some highly exceptional cases that we refer to as 'ergodic', the non-closed generators always have this (densely 2-torus-filling) geometrical property in the analytic setting.

By extending arguments we gave previously for the characterization of the Killing symmetries of higher dimensional, stationary black holes we prove that analytic, 4-dimensional, vacuum spacetimes with such (non-ergodic) compact Cauchy horizons always admit (at least) two independent, commuting Killing vector fields of which a special linear combination is horizon generating. We also discuss the *conjectures* that every such spacetime with an *ergodic* horizon is trivially constructable from the flat Kasner solution by making certain 'irrational' toroidal compactifications and that degenerate compact Cauchy horizons do not exist in the analytic case. This talk is based on joint work with Vince Moncrief.

Daniel Pollack (University of Washington)

Title: Spacetime Topology and Singularities

Abstract: We will survey a number of results connecting the topology of spacetime with the presence or absence of Marginally Outer Trapped Surfaces (MOTS) in initial data sets. This is an area of research which has been guided by the results of Greg Galloway and his collaborators.

Richard Schoen (University of California, Irvine)

Title: "Quasi-local mass and comparison geometry"

Abstract: Quasi-local mass is intended to be a quantitative measure of the complexity of a finite spatial region in a spacetime. This complexity can be thought of as a comparison with the flat Minkowski space or in the simplest Riemannian case with the euclidean space. Thus it may be thought of as comparison geometry for scalar curvature. In the past few years M. Gromov has taken a more traditional comparison approach for scalar curvature and has found theorems and conjectures for polyhedral domains in three manifolds of non-negative scalar curvature. There is a substantial gap between the quasi-local mass results and the comparison geometry results. In this lecture we will describe both sides and discuss possible ways this gap might be lessened.

Mu-Tao Wang (Columbia University)

Title: Quasilocal mass at null infinity

Abstract: An observer of an astronomical event is situated at future null infinity, where light rays emitted from the source approach. Mathematically, null infinity corresponds to the portion of the spacetime boundary defined by equivalence classes of null geodesics. But what can we observe at null infinity? In the talk, I shall discuss new results about the limit of quasilocal mass at null infinity and how this quantity should be interpreted as quasilocal radiation. This is based on joint work with Po-Ning Chen, Ye-Kai Wang, and Shing-Tung Yau.

Eric Woolgar (University of Alberta)

Title: Conditions of curvature-dimension type in relativity and Lorentzian geometry

Abstract: Curvature-dimension inequalities are modifications of a Ricci curvature bound or, in the language of relativity, an energy condition. They have proved useful in applications of Fourier analysis to diffusion processes. As tools to prove theorems in Riemannian geometry and general relativity, they are often as powerful as the usual Ricci curvature bounds and can yield new results. Applications include static Einstein metrics, near-extremal-horizon geometry, and scalar-tensor gravity. I will discuss an application of a Riemannian curvature-dimension bound to horizon topology, and use Lorentzian curvature-dimension bounds to prove some singularity theorems and splitting theorems. These are all subjects in which Greg Galloway has had huge influence. Parts of the talk are based on joint work with Marcus Khuri, Will Wylie, and, of course, Greg Galloway.

* Dr. Christina Sormani and Dr. Lars Andersson separately informed us they had to miss the conference due to emergencies.

Short Talks

Melanie Graf (University of Vienna)

Title: Rigidity of asymptotically $AdS_2 \times S^2$ spacetimes

Abstract: The product spacetime $AdS_2 \times S^2$ arises, e.g., as the near horizon geometry of the extremal Reissner-Nordstrom solution, and for that reason it has been studied in connection with the AdS/CFT correspondence. Following recent work with Greg Galloway I will look at asymptotically $AdS_2 \times S^2$ spacetimes (defined via the asymptotic behavior of the metric) that obey the null energy condition (or a certain averaged version thereof). We show that any such spacetime must contain two continuous transversal foliations by totally geodesic null hypersurfaces intersecting in isometric, totally geodesic round 2-spheres. At the end of my talk I will briefly touch upon current work in progress toward a conformal definition of "asymptotically $AdS_2 \times S^2$ ends" involving something like a 'singular scri'.

Eleni Kontou (University of York)

Title: Quantum strong energy inequality and the Hawking singularity theorem

Abstract: Hawking's singularity theorem concerns matter obeying the strong energy condition (SEC), which means that all observers experience a nonnegative effective energy density (EED), thereby guaranteeing the timelike convergence property. However, for both classical and quantum fields, violations of the SEC can be observed in some of the simplest of cases, like the massive Klein-Gordon field. Therefore there is a need to develop theorems with weaker restrictions, namely energy conditions averaged over an entire geodesic and quantum inequalities, weighted local averages of energy densities. We have derived lower bounds of the EED in the presence of both classical and quantum scalar fields allowing nonzero mass and nonminimal coupling to the scalar curvature. In the quantum case these bounds take the form of a set of state-dependent quantum energy inequalities valid for the class of Hadamard states. Finally, we discuss how these lower bounds are applied to prove Hawking-type singularity theorems asserting that, along with sufficient initial contraction at a compact Cauchy surface, the spacetime is future timelike geodesically incomplete. Talk is based on: DOI:10.1007/s10714-018-2446-5 (https://link.springer.com/article/10.1007%2Fs10714-018-2446-5), arXiv:1809.05047 (https://arxiv.org/abs/1809.05047) and a manuscript in preparation.

Eric Ling (University of Miami)

Title: Using timelike completeness to prove inextendibility of spacetimes in low regularity

Abstract: Motivated by questions arising from the strong cosmic censorship conjecture, there has been an emerging interest in low regularity aspects of general relativity. In particular it is of interest to determine which spacetimes can be isometrically embedded into a larger spacetime with a C^0 metric. It is a classical result that any timelike complete spacetime is inextendible as a spacetime with a smooth (at least C^2) metric. In this talk I will show any timelike complete and globally hyperbolic spacetime is inextendible as a spacetime with a C^0 metric. This is joint work with Greg Galloway and Jan Sbierski. Then I will show that the global hyperbolicity assumption is unnecessary for the class of Lipschitz metrics. This is joint work with Melanie Graf.

Armando Cabrera Pacheco (University of Tübingen)

Title: On Bartnik mass estimates of CMC Bartnik data

Abstract: The problem of computing Bartnik's quasi-local mass for a domain can be rephrased as an extension problem for Bartnik data, i.e., 2-surfaces with mean curvature H. Recently, C. Mantoulidis and R. Schoen constructed asymptotically flat extensions of Bartnik data with $H = 0$ allowing them to compute their Bartnik mass. We will describe how to adapt their ideas to construct extensions and obtain estimates for the Bartnik mass of Bartnik data with H a positive constant. This talk is based on a joint paper with C. Cederbaum, S. McCormick, and P. Miao.

Didier A. Solis Gamboa (Universidad Autonoma de Yucatan)

Title: Towards a classification of Einstein null hypersurfaces in RW-spacetimes

In this talk we present the notion of a screen quasi-conformal null hypersurface and provide examples of its use in classifying isoparametric and Einstein null hypersurfaces in GRW-spacetimes. This is joint work with M. Navarro (UADY) and O. Palmas (UNAM)

Carlos Vega (Binghamton University)

Title: Splitting Spacetime

Abstract: In this short talk, I will discuss joint work with G. Galloway on spacetime rigidity and splitting geometry.