MTH 309

Additional Problems for Sec. 2.3

1. Let $g : A \to B$ and $f : B \to C$ be functions. Justify your answer on each of the following.

   (a) If $f \circ g$ is one to one, does it follow that g is one to one?

   (b) If $f \circ g$ is one to one, does it follow that f is one to one?

   (c) If $f \circ g$ is onto, does it follow that g is onto?

   (d) If $f \circ g$ is onto, does it follow that f is onto?

2. Let $S = \{1, 2, \ldots, n\}$.

   (a) Find a bijection from the set of subsets of S of even cardinality to the set of subsets of S of odd cardinality. (Describe the domain and codomain with set builder notation and give the rule.)

   (b) Can you conclude from (a) that the number of subsets of S of even cardinality equals the number of subsets of S of odd cardinality?

   (c) Find a formula for the number of subsets of S that have an even number of elements.