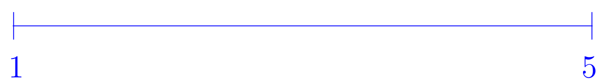
Supplemental Examples and Excercises:

Left and Right Hand Sums

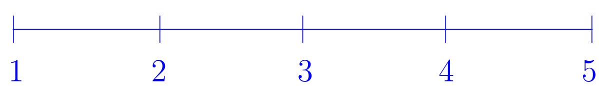
Example: Find the left and right hand sums for $f(x) = x^2 + 1$ over the interval $1 \leq x \leq 5$ using $n = 4$ first, then using $n = 8$. Include sketches each time.

Solution: We will first find LHS and RHS using $n = 4$.

Hence, we take our interval:



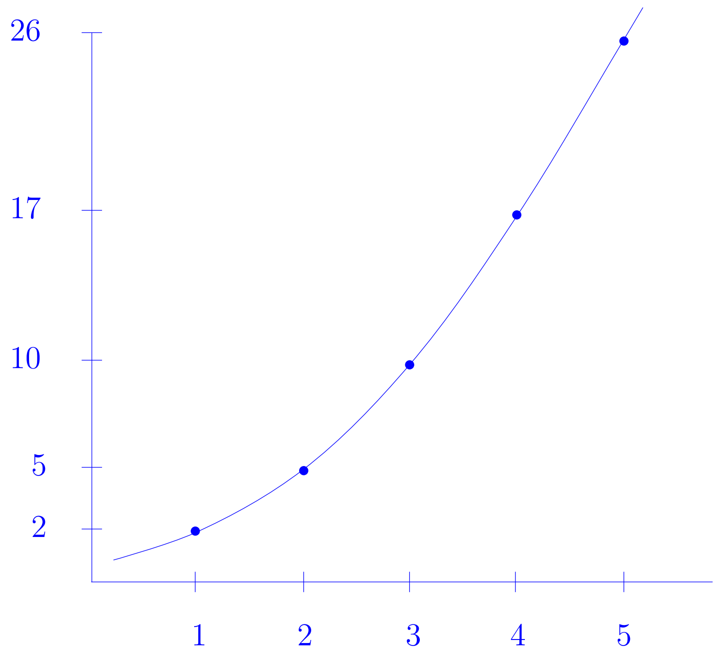
and chop it into $n = 4$ equal pieces:



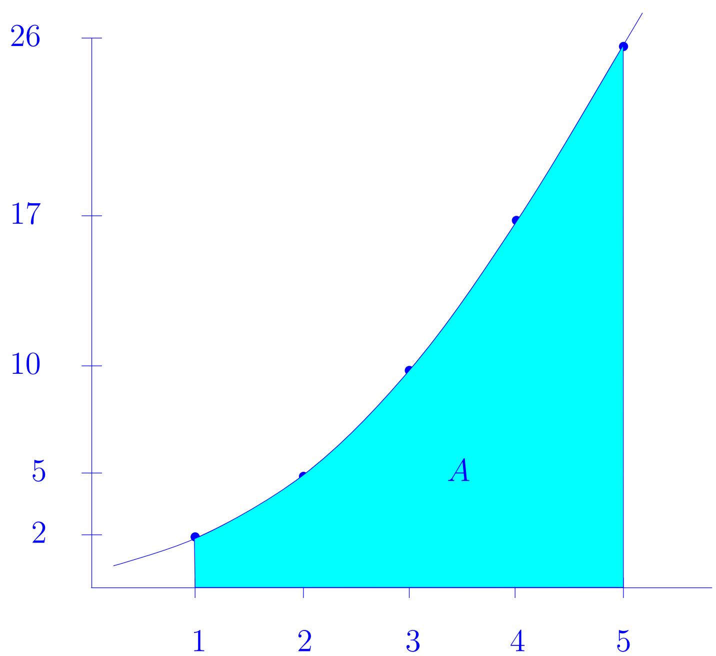
We plot these points in a table:

x	1	2	3	4	5
$f(x) = x^2 + 1$	2	5	10	17	26

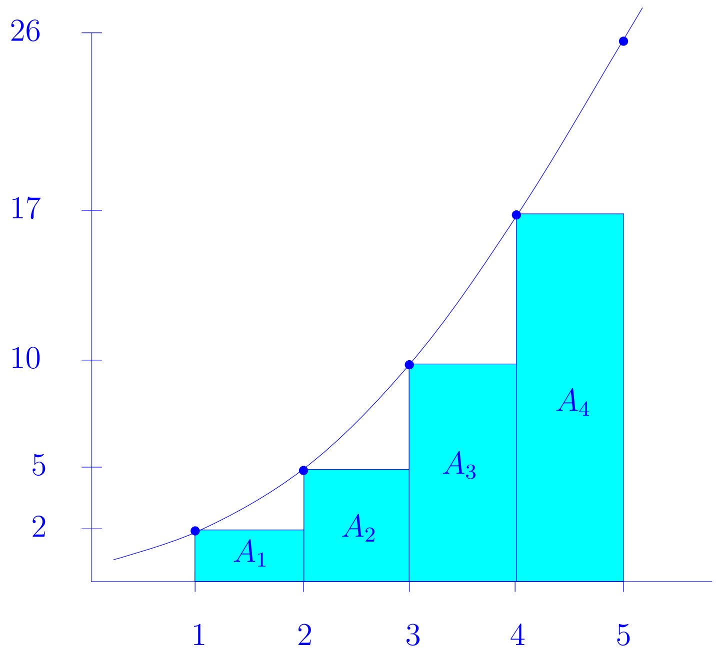
... and on an x-y plane, sketching the graph of $f(x) = x^2 + 1$:



Note that the area we will approximate with the LHS and RHS is:



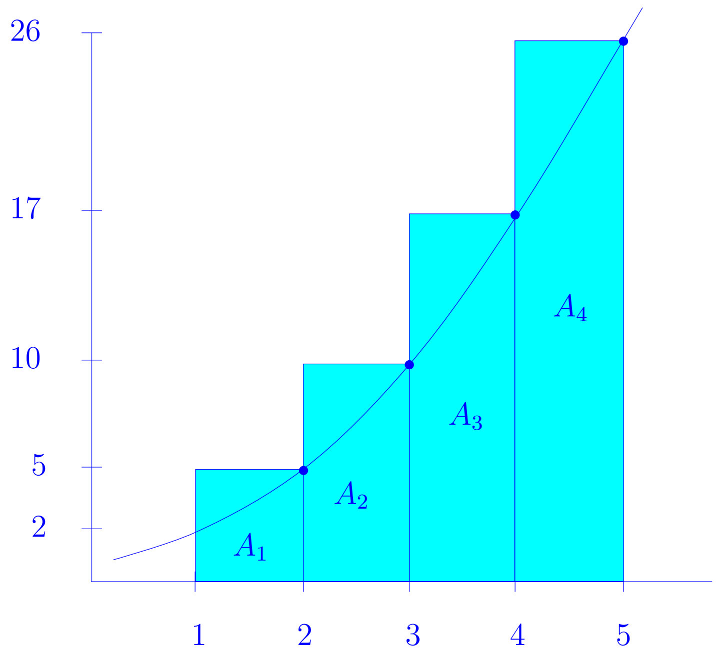
To form the left hand sum (LHS), we draw a rectangle over each piece, with the upper *left* corners touching the graph:



Hence, we have:

$$\begin{aligned}
\text{LHS} &= A_1 + A_2 + A_3 + A_4 \\
&= (2 \cdot 1) + (5 \cdot 1) + (10 \cdot 1) + (17 \cdot 1) \\
&= 2 + 5 + 10 + 17 \\
&= 34
\end{aligned}$$

To form the right hand sum (RHS), we draw a rectangle over each piece, with the upper *right* corners touching the graph:



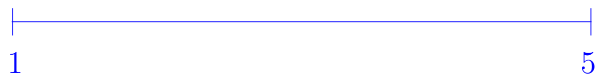
Hence, we have:

$$\begin{aligned} \text{RHS} &= A_1 + A_2 + A_3 + A_4 \\ &= (5 \cdot 1) + (10 \cdot 1) + (17 \cdot 1) + (26 \cdot 1) \\ &= 5 + 10 + 17 + 26 \\ &= 58 \end{aligned}$$

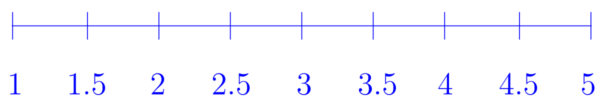
So, using $n = 4$, we get LHS = 34 and RHS = 58.

Now we find LHS and RHS using $n = 8$.

Hence, we take our interval:



and chop it into $n = 8$ equal pieces:



Note that the width of the pieces is now 0.5.

We plot these points in a table:

x	1	1.5	2	2.5	3	3.5	4	4.5	5
$x^2 + 1$	2	3.25	5	7.25	10	13.25	17	21.25	26

As before, plot these points, and **redraw** LHS and RHS, now using 8 rectangles of width 0.5. You should get LHS = 39.5 and RHS = 51.5.

... Of course, by now we now how to find the area A exactly using the Fundamental Theorem of Calculus. What is it? (Hint: Remember, the first step is to find an antiderivative of $f(x) = x^2 + 1$.)