Supplemental Examples and Excercises:
Antiderivatives and Definite Integrals

Example: Use the shortcut rules to find each antiderivative.

a) $\int 4\,dx$

Solution: When finding the antiderivative of 4, the question is:

4 is the derivative of *what function?*

$$\int 4 = 4x + C$$

b) $\int (x+3)\,dx$

Solution: Here the question is:

$x + 3$ is the derivative of *what function?*

Using the shortcut rules, we have:

$$\int (x+3) \;=\; \int x \;+ \int 3 \;=\; \frac{x^2}{2} + 3x + C$$

c) $\int (x^2 + 5x + 7)\,dx$

Solution:

$$\int (x^2+5x+7) \;=\; \int x^2 \;+\; 5\int x \;+ \int 7 \;=\; \frac{x^3}{3} \;+\; 5\cdot\frac{x^2}{2} \;+\; 7x \;+\; C$$

Example: Use the Fundamental Theorem of Calculus to find each definite integral.

a) $\int_2^7 4\,dx$

Solution: Recall that, for positive functions, the definite integral $\int_a^b f(x)\,dx$ is the area under $f(x)$, between $x = a$ and $x = b$. The Fundamental Theorem of Calculus (FTC) says:

$$\int_a^b f(x) = F(x)\Big|_a^b = F(b) - F(a),$$

where $F(x)$ is *any* antiderivative of $f(x)$. (Hence, we can always take '$C = 0$'.)

Thus, in this case, FTC says:

$$\int_2^7 4 = \Big(4x\Big|_2^7 = 4(7) - 4(2) = 28 - 8 = 20$$

b) $\int_1^2 (x+3)\,dx$

Solution: In this case, FTC says:

$$\int_1^2 (x+3) = \left(\frac{x^2}{2} + 3x\right|_1^2 = \left(\frac{(2)^2}{2} + 3(2)\right) - \left(\frac{(1)^2}{2} + 3(1)\right) = 8 - 3.5 = 4.5$$

c) $\int_0^1 (x^2 + 5x + 7)\,dx$

Solution: By plugging in the endpoints and subtracting as above, you may check the final answer below:

$$\int_0^1 (x^2 + 5x + 7) = \left(\frac{x^3}{3} + 5 \cdot \frac{x^2}{2} + 7x\right|_0^1 \approx 9.83$$

Exercise 1: Find each antiderivative.

a) $\int 7\,dx$

b) $\int (7x - x^2)\,dx$

c) $\int (6x^2 - 4x + 8)\,dx$

d) $\int \dfrac{2}{\sqrt{x}}\,dx$

Exercise 2: Find each definite integral.

a) $\int_0^4 7\,dx$

b) $\int_1^3 (7x - x^2)\,dx$

c) $\int_0^1 (6x^2 - 4x + 8)\,dx$

d) $\int_1^4 \dfrac{2}{\sqrt{x}}\,dx$

e) $\int_2^4 (9 + x)\,dx$

f) $\int_0^1 (1 - x^6)\,dx$

Answers: (Try before looking!)

1: a) $7x + C$ b) $7 \cdot \dfrac{x^2}{2} - \dfrac{x^3}{3} + C$ c) $2x^3 - 2x^2 + 8x + C$ d) $4\sqrt{x} + C$

2: a) 28 b) 16.5 c) 8 d) 4 e) 24 f) $\dfrac{6}{7}$