Antiderivative Rules and the Fundamental Theorem of Calculus

Given a function $f(x)$, we say that $F(x)$ is an *antiderivative* of f if

$$F'(x) \;=\; f(x)$$

We also write the antiderivative of f as:

$$\int f(x)\,dx$$

We have the following rules for antiderivatives. Let c, k, and n be constants ($n \neq -1$).

Antiderivative of a constant:

$$\int k = kx + C$$

Antipower rule:

$$\int x^n = \frac{x^{n+1}}{n+1} + C$$

Sum/difference rule:

$$\int (f \pm g) = \int f \pm \int g$$

Constant multiple rule:

$$\int (c \cdot f) = c \int f$$

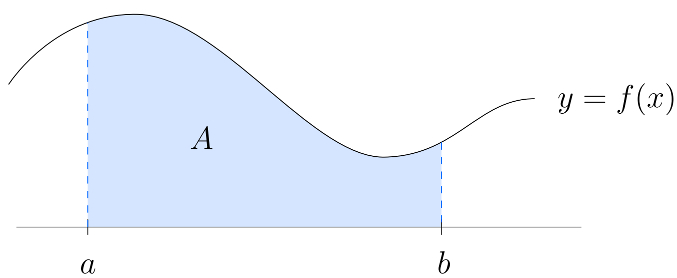
... where C stands for an arbitrary constant.

The Fundamental Theorem of Calculus:

Let f be positive on the interval $[a, b]$.

The *definite integral*, $\int_a^b f(x)dx$, may be defined as:

$$\int_a^b f(x)\,dx \;=\; A \;=\; \text{the area under } f,\ \text{from } x = a \text{ to } x = b$$



The Fundamental Theorem of Calculus tells us how to find such an area:

$$\int_a^b f(x)\,dx \;=\; F(x)\Big|_a^b = F(b) - F(a)$$

where $F(x)$ is any antiderivative of $f(x)$, i.e., any function satisfying $F'(x) = f(x)$.