Answers/Solutions to Assigned Even Problems

Section 2.1:

In Exercises 1-12, compute the derivative of the given function and find the slope of the line that is tangent to its graph for the specified value of the independent variable.

2. $f(x) = -3$; $x = 1$.

Solution: Recall that, at this section, we have no 'shortcut rules' for derivatives, and must use the (limit) definition;

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

But, in this case, f is unusually simple; $f(x) = -3$. This means

$$f(\text{anything}) = -3$$

In particular, $f(x+h) = -3$. Hence, we have:

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{[-3] - [-3]}{h} \\ &= \lim_{h \to 0} \frac{0}{h} \\ &= \lim_{h \to 0} (0) \\ &= 0 \end{aligned}$$

Indeed, the graph of $f(x) = -3$ is a horizontal line, so the slope at each point is 0.

4. $f(x) = 2 - 7x$; $x = -1$

Solution: You can run through the definition, but again, this function is linear. It's graph is a line, with slope $m = -7$. So the slope at each point is -7. In particular, $f'(x) = -7$.

6. $f(x) = x^2 - 1$; $x = -1$.

Solution: Now f is a 'curved' function. We apply the definition of the derivative:

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{[(x+h)^2 - 1] - [x^2 - 1]}{h} \\
&= \lim_{h\to 0} \frac{[x^2 + 2xh + h^2 - 1] - [x^2 - 1]}{h} \\
&= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 1 - x^2 + 1}{h} \\
&= \lim_{h\to 0} \frac{2xh + h^2}{h} \\
&= \lim_{h\to 0} \frac{h(2x + h)}{h} \\
&= \lim_{h\to 0} (2x + h) \\
&= 2x + (0) \\
&= 2x
\end{aligned}$$

The derivative is $f'(x) = 2x$. This gives:

$$\text{the slope of the tangent at } x = -1 \quad \text{is} \quad m = f'(-1) \ = 2(-1) = -2$$

2.1.14. **Answer:** $f'(x) = 0$, $y = 3$

2.1.16. **Answer:** $f'(x) = 3$, $y = 3x$

2.1.18. **Answer:** $f'(x) = -6x$, $y = -6x + 5$

2.2.2 **Answer:** $y' = 0$

2.2.4 **Answer:** $y' = -2$

2.2.6 **Answer:** $y' = \frac{7}{3}x^{\frac{4}{3}}$

2.2.8 **Answer:** $y' = 1.2x^{-2.2}$

2.2.10 **Answer:** $4\pi r^2$ (The derivative of the volume of a sphere is its surface area.)

2.2.12 **Answer:** First rewrite: $y = 2x^{\frac{3}{4}}$. Thus, $y' = 2 \cdot \frac{3}{4} \cdot x^{-\frac{1}{4}} = \frac{3}{2}x^{-\frac{1}{4}}$.

2.2.14 **Answer:** First rewrite: $y = 3t^{-2}$. Thus, $y' = -6t^{-3}$.

2.2.16 **Answer:** $y' = 15x^4 - 12x^2 + 9$

2.2.18 **Answer:** $f'(x) = 2x^7 - 3x^5 - 1$

2.2.20 **Answer:** $f'(u) = 0.28u^3 - 3.63u^2 + 3$

2.2.22 **Ans:** First rewrite: $y = 3x^{-1} - 2x^{-2} + \frac{2}{3}x^{-3}$. Hence, $y' = -3x^{-2} + 4x^{-3} - 2x^{-4}$.

2.2.24 **Answer:** First rewrite: $f(x) = 2t^{\frac{3}{2}} + 4t^{-\frac{1}{2}} - \sqrt{2}$. Then:

$$f'(x) = 2 \cdot \frac{3}{2} \cdot t^{\frac{1}{2}} + 4 \cdot \left(-\frac{1}{2}\right) \cdot t^{-\frac{3}{2}} = 3t^{\frac{1}{2}} - 2t^{-\frac{3}{2}}$$

2.2.26 **Answer:** Rewritten, $y = -7x^{-1.2} + 5x^{2.1}$. Hence, $y' = 8.4x^{-2.2} + 10.5x^{1.1}$.

2.2.28 **Answer:** $y' = 5x^4 - 18x^2 + 14x$

2.2.30 **Solution:** We want the equation of the tangent line $y = x^5 - 3x^3 - 5x + 2$, at the point $(1, -5)$.

Recall that, given a function $f(x)$, the tangent line to f at $x = a$:

i) goes through the point $(a, f(a))$, and

ii) has slope $m = f'(a)$.

i) In this case, part i) is already done for us; Note that

$$f(1) = (1)^5 - 3(1)^3 - 5(1) + 2 = -5$$

ii) To get the slope, we first find y', (i.e., $f'(x)$), then plug in $x = 1$:

$$y' = 5x^4 - 9x^2 - 5$$

Hence, the slope at $x = 1$ is:

$$m = y'(1) = 5(1)^4 - 9(1)^2 - 5 = -9$$

To get the equation, we can plug into 'point-slope form':

$$\begin{aligned} y - (-5) &= (-9)(x - 1) \\ y + 5 &= -9x + 9 \\ y &= -9x + 4 \end{aligned}$$

2.2.32 **Answer:** $y' = \frac{3}{2}\sqrt{x} - 2x - 32x^{-3}$. Hence, $y'(4) = -5.5$, and the tangent at $x = 4$ is

$$y = -5.5x + 23.75$$

2.2.36 **Solution:** We want the equation of the tangent line $f(x) = x^4 - 3x^3 + 2x^2 - 6$ at $x = 2$.

Again, recall that, given a function $f(x)$, the tangent line to f at $x = a$:

i) goes through the point $(a, f(a))$, and

ii) has slope $m = f'(a)$.

i) We have:

$$f(2) = (2)^4 - 3(2)^3 + 2(2)^2 - 6 = 16 - 24 + 8 - 6 = -6$$

Hence, the tangent at $x = 2$ goes through the point $(2, f(2)) = (2, -6)$.

ii) To get the slope, we first find $f'(x)$, then plug in $x = 2$:

$$f'(x) = 4x^3 - 9x^2 + 4x$$

Hence, the slope at $x = 2$ is:

$$m = f'(2) = 4(2)^3 - 9(2)^2 + 4(2) = 32 - 36 + 8 = 4$$

To get the equation, we can plug into 'point-slope form':

$$\begin{aligned} y - (-6) &= (4)(x - (2)) \\ y + 6 &= 4x - 8 \\ y &= 4x - 14 \end{aligned}$$

2.2.38 **Answer:** $f'(x) = 3x^2 + \dfrac{1}{2\sqrt{x}}$. Hence, $f'(4) = 48.25$, and the tangent at $x = 4$ is

$$y = 48.25x - 127$$

2.2.40 **Answer:** After some work, we get: $f'(x) = \frac{3}{2}\sqrt{x} - 1$. Hence, $f'(4) = 2$, and the tangent at $x = 4$ is

$$y = 2x - 4$$

2.3.2 **Answer:** (not simplified) $f'(x) = (1)(1 - 2x) + (x - 5)(-2)$

2.3.4 **Answer:** (not simplified) $y = 400[(-2x)(3x - 2) + (15 - x^2)(3)]$

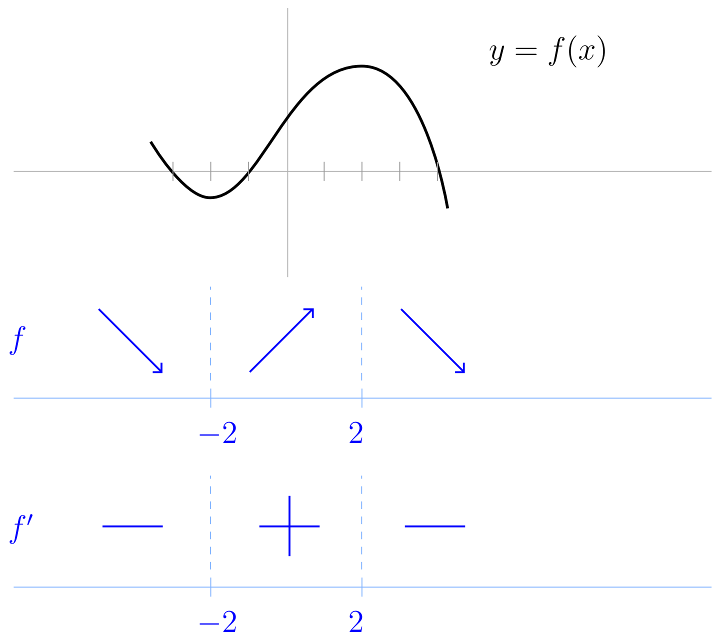
2.3.6 **Answer:** (not smp'd) $f'(x) = -3[(15x^2 - 2)(\sqrt{x} + 2x) + (5x^3 - 2x + 5)(\frac{1}{2}x^{-\frac{1}{2}} + 2)]$

2.3.8 **Answer:** (not smp'd) $f'(x) = \dfrac{2(5x + 4) - (2x - 3)(5)}{(5x + 4)^2}$

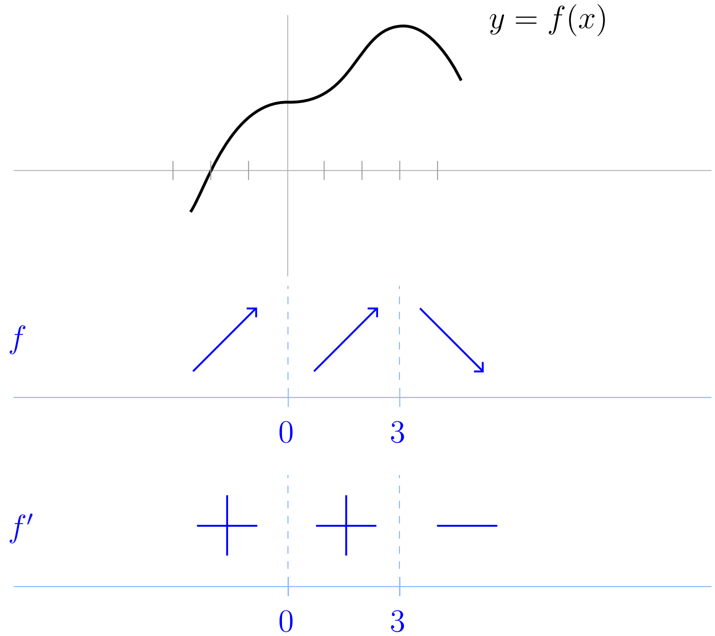
2.3.10 **Answer:** $f'(x) = \dfrac{-1}{(x - 2)^2}$

2.3.12 **Answer:** (not smp'd) $f'(x) = \dfrac{(2t)(1 - t^2) - (t^2 + 1)(-2t)}{(1 - t^2)^2}$

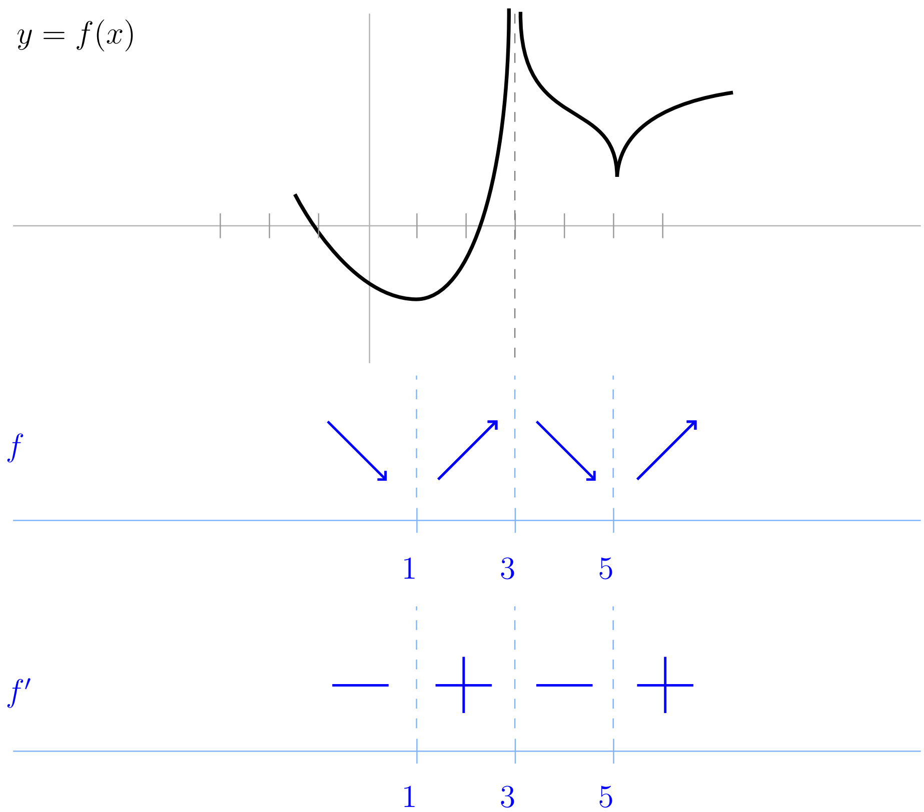
3.1.1



3.1.2



3.1.4

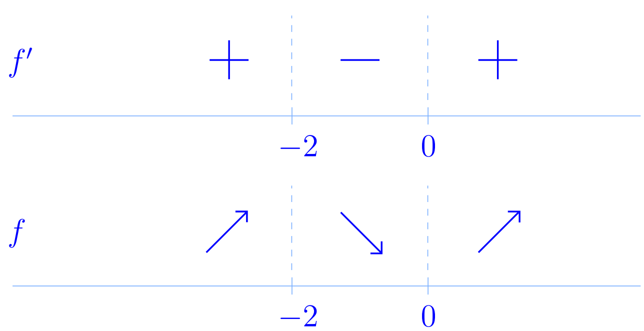


Hence, f' is positive on $(1, 3)$ and $(5, \infty)$ and negative on $(-\infty, 1)$ and $(3, 5)$.

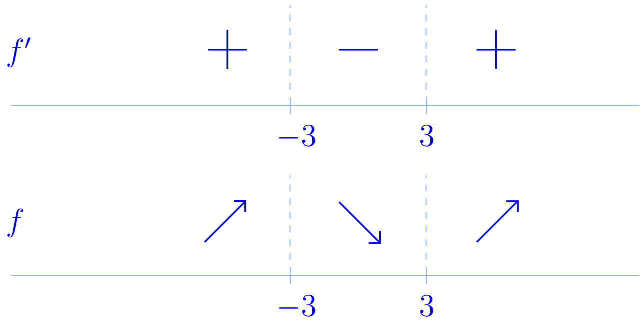
3.1.6 **Answer:** C

3.1.8 **Answer:** A

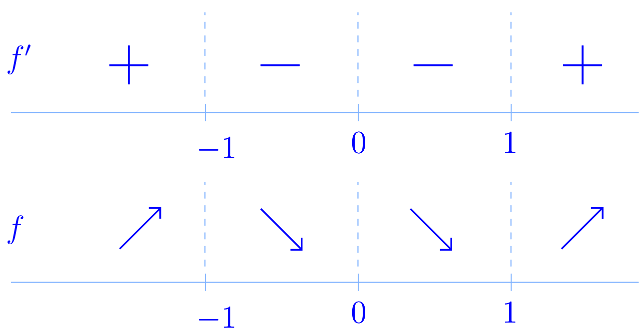
3.1.10 **Solution:** $f'(t) = 3t^2 + 6t = 3t(t + 2)$. Critical points: $t = -2, 0$



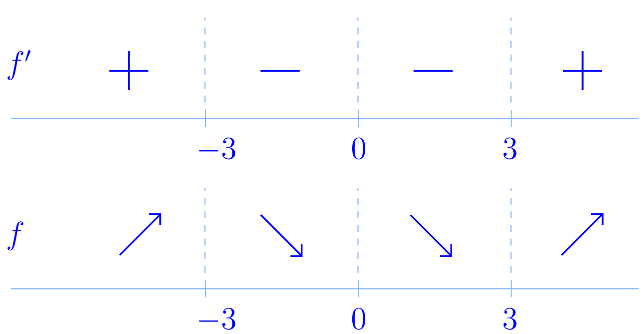
3.1.12 **Solution:** $f'(x) = x^2 - 9 = (x-3)(x+3)$. Critical points: $x = -3, 3$



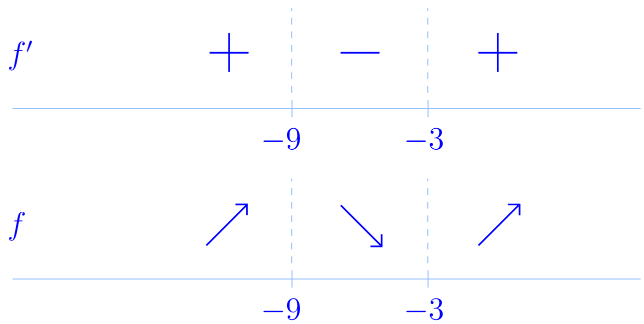
3.1.14 **Solution:** $f'(x) = 15x^4 - 15x^2 = 15x^2(x^2-1) = 15x^2(x-1)(x+1)$. Critical points: $x = -1, 0, 1$



3.1.19 **Solution:** $f'(x) = 1 - 9x^{-2} = 1 - \frac{9}{x^2} = \dfrac{x^2-9}{x^2}$. Critical points: $x = -3, 0, 3$

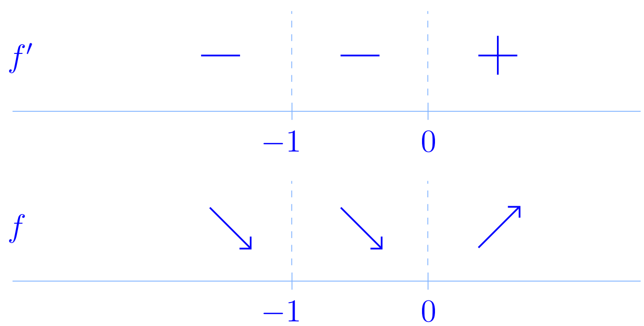


3.1.24 **Solution:** $f'(x) = 324 - 144x + 12x^2 = 12(x^2 + 12x + 27) = 12(x + 9)(x + 3)$. Critical points: $x = -9, -3$.



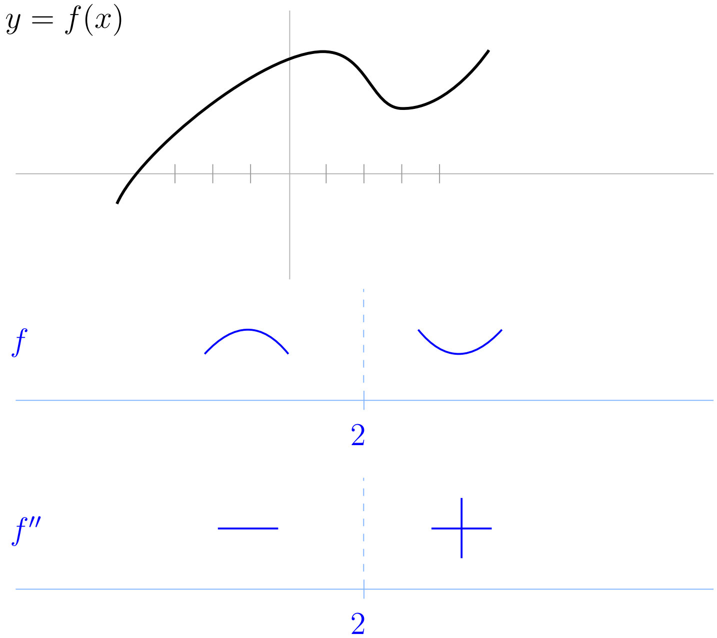
Hence, (plugging back into the original function), f has a relative maximum ('peak') at $(-9, 162)$, and a relative minimum ('valley') at $(-3, -1782)$.

3.1.26 **Solution:** $f'(x) = 60t^5 + 120t^4 + 60t^3 = 60t^3(t^2 + 2t + 1) = 60t^3(t + 1)^2$. Critical points: $x = -1, 0$.

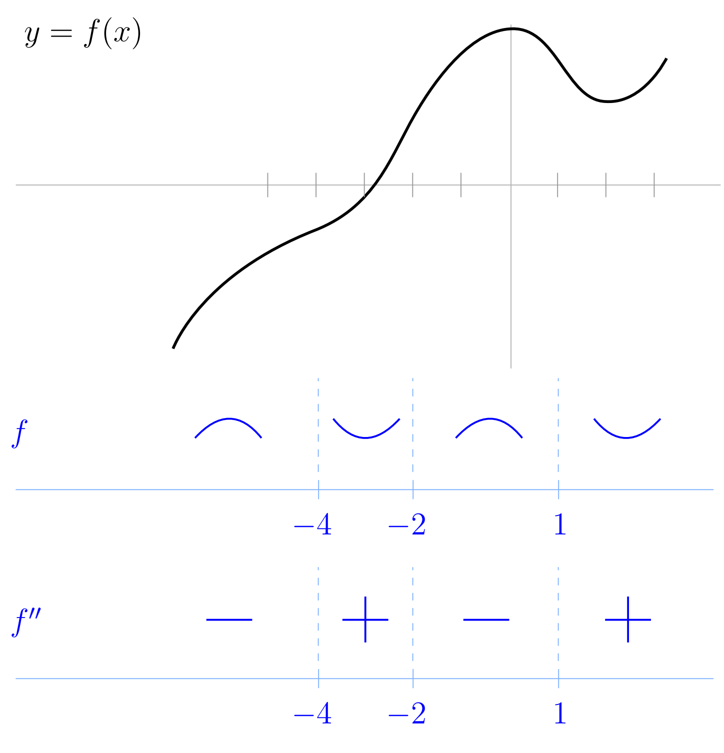


f has a relative minimum ('valley') at $(0, 3)$. The critical point $x = -1$ is neither a relative max nor relative min.

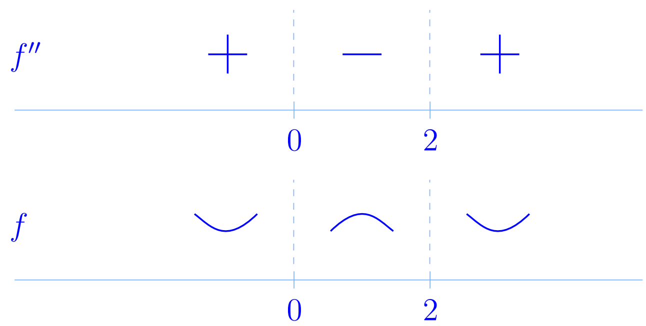
3.2.1



3.2.2

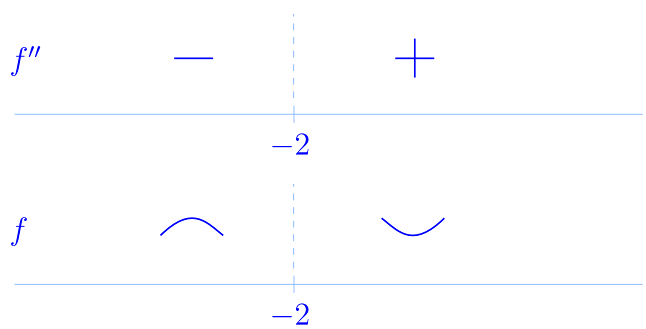


3.2.6 **Solution:** $f''(t) = 12x^2 - 24x = 12x(x-2)$. 'Critical points': $x = 0, 2$



f has inflection points at $(0, -9)$ and $(2, -5)$. (Plug into original f.)

3.2.8 **Solution:** $f''(s) = 6s + 12 = 6(s+2)$. 'Critical points': $x = -2$

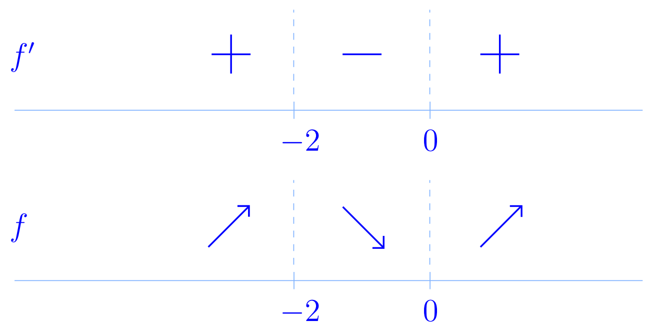


f has an inflection point at $(-2, -2)$. (Plug into original f.)

3.2.14 **Solution:**

I. Make a sign chart for f'.

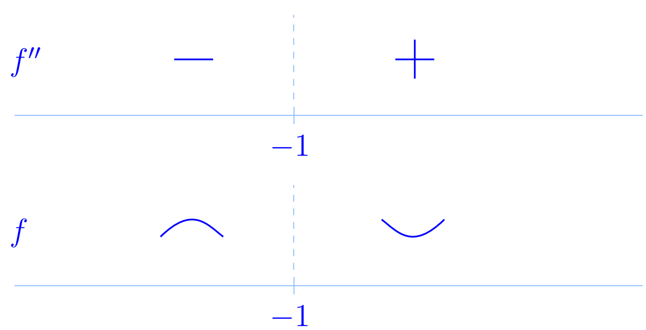
We have $f'(x) = 3x^2 + 6x = 3x(x+2)$. Critical points: $x = -2, 0$



Hence, f is increasing on $(-\infty, -2)$ and $(0, \infty)$ and decreasing on $(-2, 0)$. f has a relative maximum at $(-2, 5)$ and a relative minimum at $(0, 1)$.

II. Make a sign chart for f''.

We have $f''(x) = 6x + 6 = 6(x+1)$. Critical points: $x = -1$



Hence, f is concave down on $(-\infty, -1)$ and concave up on $(-1, \infty)$. f has an inflection point at $(-1, 3)$.