Answers/Solutions to Assigned Even Problems:

1.1.34: Find the difference quotient for $f(x) = 2x + 3$.

Solution: Recall that the *difference quotient* of a function is:

$$\frac{f(x+h) - f(x)}{h}$$

For $f(x) = 2x + 3$ we have:

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{[2(x+h)+3] - [2x+3]}{h} \\
&= \frac{2x + 2h + 3 - 2x - 3}{h} \\
&= \frac{\cancel{2x} + 2h + \cancel{3} - \cancel{2x} - \cancel{3}}{h} \\
&= \frac{2h}{h} \\
&= 2
\end{aligned}$$

1.1.36: Find the difference quotient for $f(x) = x^2$. **Answer:** $2x + h$.

1.3.10: **Answer:** x-intercept = -4, y-intercept = 2.5, slope = $\dfrac{2.5}{4} = 0.625$

1.3.12: **Answer:** x-intercept = -2.5, y-intercept = -3, slope = $\dfrac{-3}{2.5} = -1.2$

1.3.22: Find the equation of the line through $(-1, 2)$ with slope $\frac{2}{3}$. **Ans:** $y = \frac{2}{3}x + \frac{8}{3}$

1.3.24: Find the equation of the line through $(0, 0)$ with slope 5. **Answer:** $y = 5x$

1.3.26: Find the equation of the line through $(2, 5)$ which is parallel to the y axis. **Answer:** $x = 2$ (parallel to the y axis means vertical)

1.3.28: Find the equation of the line through $(2,5)$ and $(1,-2)$.

Solution: First find the slope:

$$m = \frac{(-2)-(5)}{(1)-(2)} = \frac{-7}{-1} = 7$$

Method 1: Using 'point-slope' form:

Now that we have the slope, we can use point-slope form with $(x_0, y_0) = (2,5)$ and $m = 7$:

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - (5) &= (7)(x - (2)) \\ y - 5 &= 7x - 14 \\ y &= 7x - 9 \end{aligned}$$

Method 2: Using 'slope-intercept' form:

Since we know the slope, $m = 7$, slope-intercept form gives:

$$\begin{aligned} y &= mx + b \\ \implies \quad y &= 7x + b \end{aligned}$$

We can then find b by plugging in either point into this equation. For example, plugging in $(1,-2)$, we get:

$$\begin{aligned} (-2) &= 7(1) + b \\ \implies \quad b &= -9 \end{aligned}$$

Hence, we get:

$$y = 7x - 9$$

1.3.30: Find the equation of the line through $(-2, 3)$ and $(0, 5)$. **Answer:** $y = x + 5$

1.3.32: Find the equation of the line through $(1, 5)$ and $(1, -4)$. **Answer:** $x = 1$

1.5.42: **Answer:** 3

2.1.2: Find the slope of the tangent line to $f(x) = -3$ at $x = 1$.

Solution: Recall that the slope of the tangent line at $x = 1$ is defined by:

$$\text{slope of tangent at } x = 1 \quad = \quad \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

Now, $f(x) = -3$ is a *constant* function, i.e., $f(\text{anything}) = -3$. So we have:

- $f(1) = -3$
- $f(1+h) = -3$

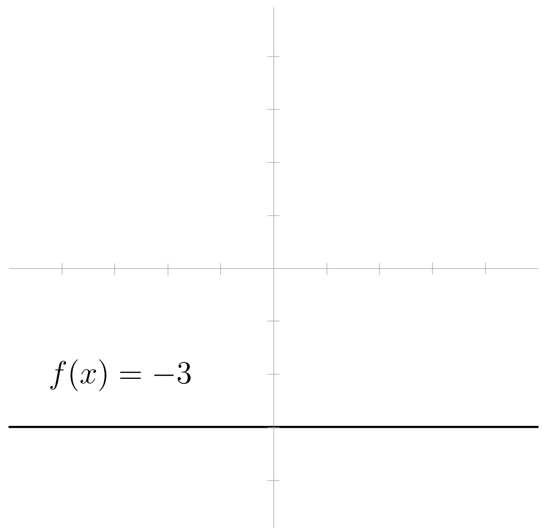
So the slope formula gives:

$$\begin{aligned} \text{slope of tangent line at } x = 1 &= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} \\ &= \lim_{h \to 0} \frac{[-3] - [-3]}{h} \\ &= \lim_{h \to 0} \left(\frac{0}{h}\right) \\ &= \lim_{h \to 0} (0) \\ &= 0 \end{aligned}$$

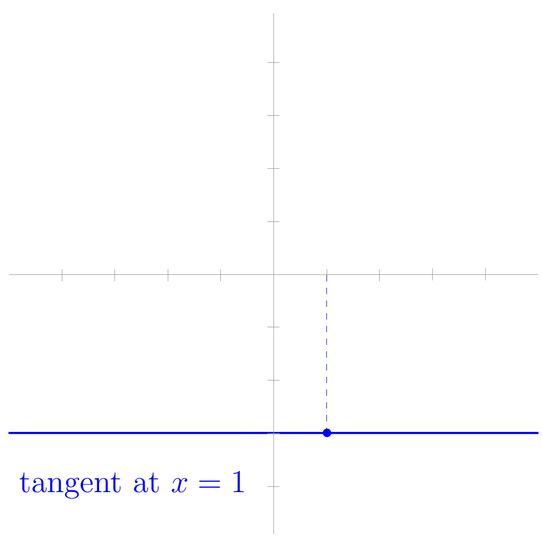
See the next page for an explanation this result.

Here's the picture for what's going on in Problem 2.1.2:

The function $f(x) = -3$ is a constant function. Hence, its graph is a horizontal line, which has slope $m = 0$.



The tangent line to this graph at $x = 1$ is the line that touches the graph at $(1, f(1))$ and lays 'flattest' against the graph there. Hence, in this case, the tangent line is the original line itself, and thus also has slope $m = 0$.



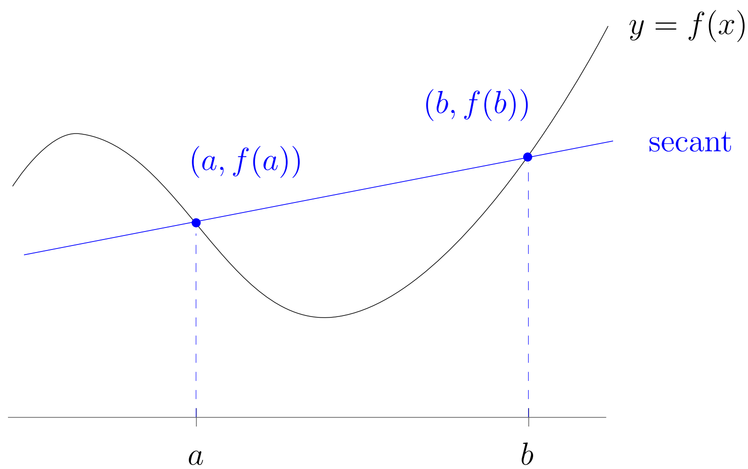
2.1.4: Find the slope of the tangent line to $f(x) = 2 - 7x$ at $x = -1$. **Answer:** -7

2.1.6: Find the slope of the tangent line to $f(x) = x^2 - 1$ at $x = -1$. **Answer:** -2

2.1.34: Let $f(x) = 2x - x^2$.

a) Compute the slope of the secant line joining the points where $x = 0$ and $x = \frac{1}{2}$.

Solution: Recall that, in general, the secant line to a function $f(x)$ through $x = a$ and $x = b$ is the (unique) line that goes through $(a, f(a))$ and $(b, f(b))$:



$$\text{slope of secant} = \frac{f(b) - f(a)}{b - a}$$

Solution: Hence, for $f(x) = 2x - x^2$, the secant line through $x = 0$ and $x = \frac{1}{2}$ is:

$$\frac{f(\frac{1}{2}) - f(0)}{\frac{1}{2} - 0} = \frac{[\,2(\frac{1}{2}) - (\frac{1}{2})^2\,] - [\,2(0) - (0)^2\,]}{\frac{1}{2}} = \frac{[\,1 - \frac{1}{4}\,] - [\,0\,]}{\frac{1}{2}} = \frac{\frac{3}{4}}{\frac{1}{2}} = \frac{3}{4} \cdot \frac{2}{1} = \frac{6}{4} = \frac{3}{2}$$

b) Use calculus to compute the slope of the tangent line to $f(x)$ at $x = 0$, and compare with the slope found in part a).

Solution: Since $f(x) = 2x - x^2$, we have:

$$\begin{aligned}
\text{slope of tangent line at } x = 0 \quad &= \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} \\
&= \lim_{h \to 0} \frac{f(h) - f(0)}{h} \\
&= \lim_{h \to 0} \frac{[2(h) - (h)^2] - [2(0) - (0)^2]}{h} \\
&= \lim_{h \to 0} \frac{2h - h^2}{h} \\
&= \lim_{h \to 0} \frac{h(2 - h)}{h} \\
&= \lim_{h \to 0} \frac{\cancel{h}(2 - h)}{\cancel{h}} \\
&= \lim_{h \to 0} (2 - h) \\
&= 2
\end{aligned}$$

(As usual, we can think of the last step as simply plugging in $h = 0$.)

Note that this is 'close' to the slope found in part a).