Buffon's Noodle

Drew Armstrong
University of Miami

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- “Comte de Buffon”
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and most importantly …

- Gambler

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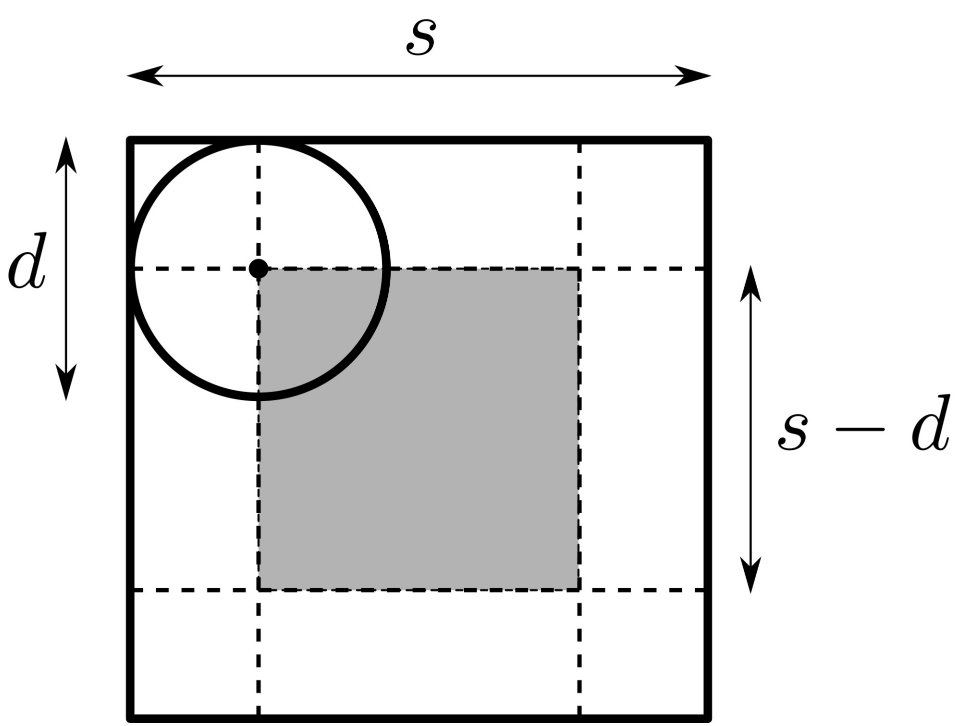
Answer: $$\left(1 - \frac{d}{s}\right)^2$$

Proof: Consider a typical tile.



Coin center lands in ...

grey = you win

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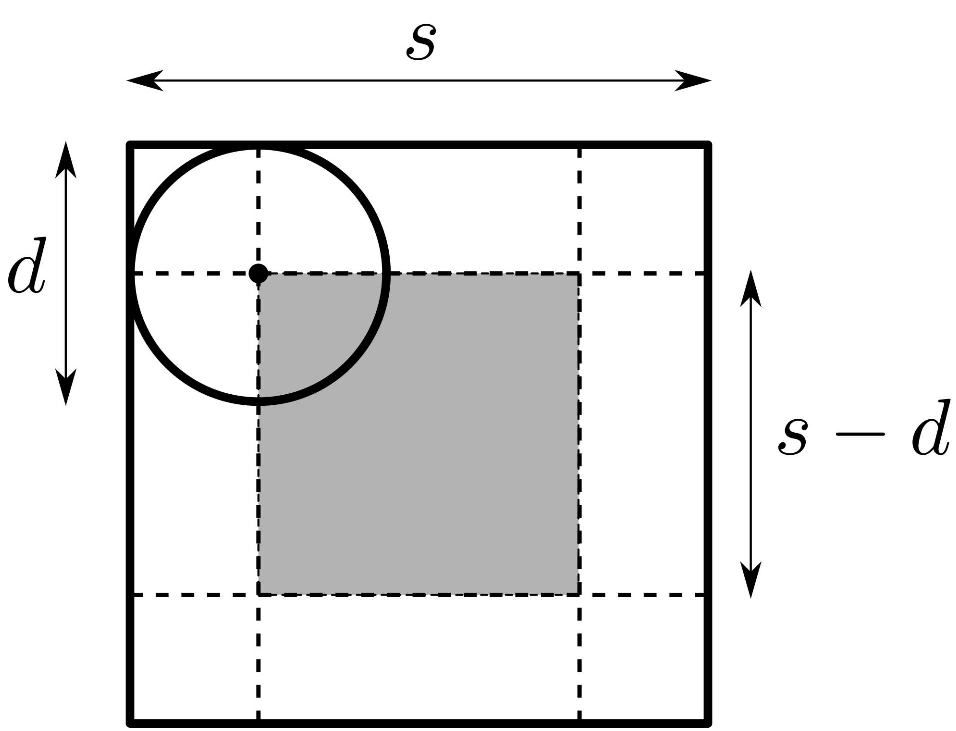

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$$P(\text{you win}) = \frac{\text{grey area}}{\text{total area}} = \frac{(s-d)^2}{s^2} = \left(1 - \frac{d}{s}\right)^2$$

$\square$

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- Instead, throw a **needle** (or a *baguette*) of length d.
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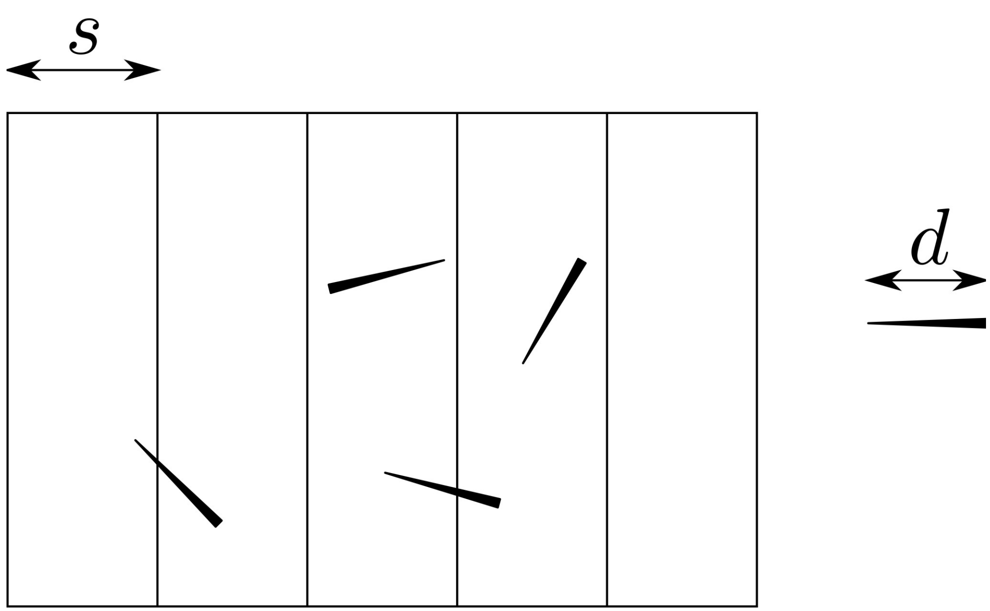

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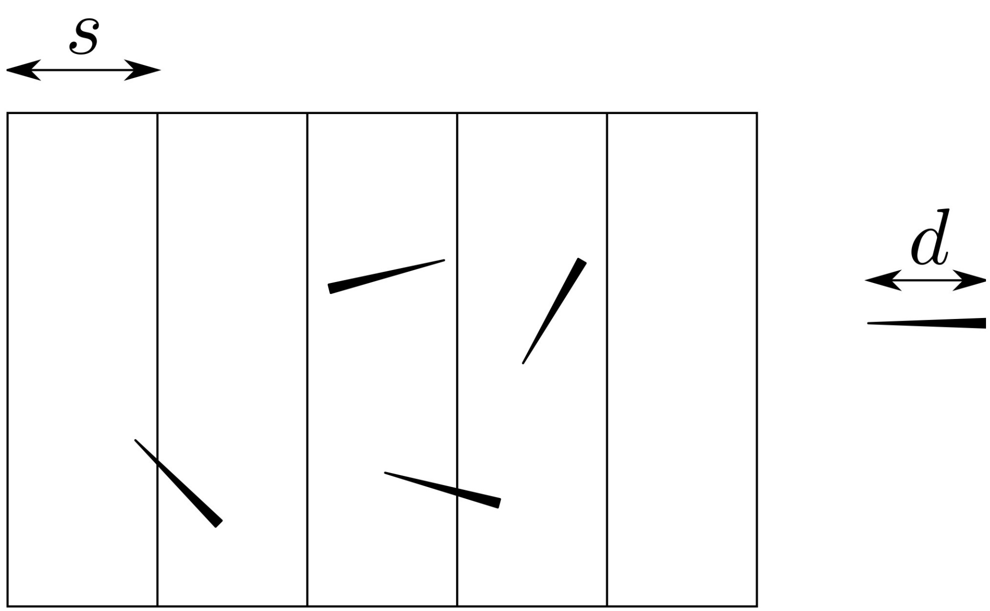

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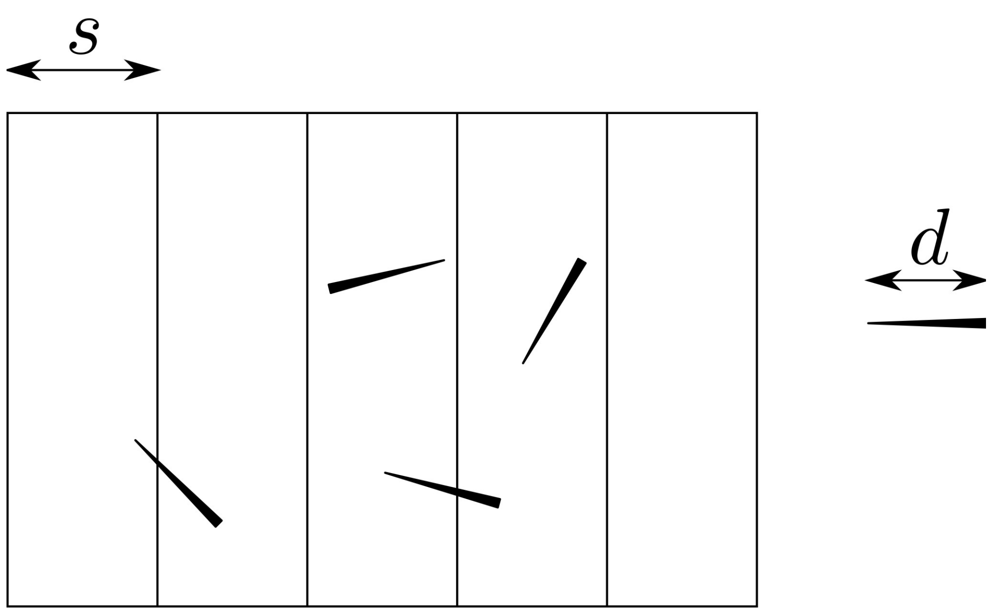

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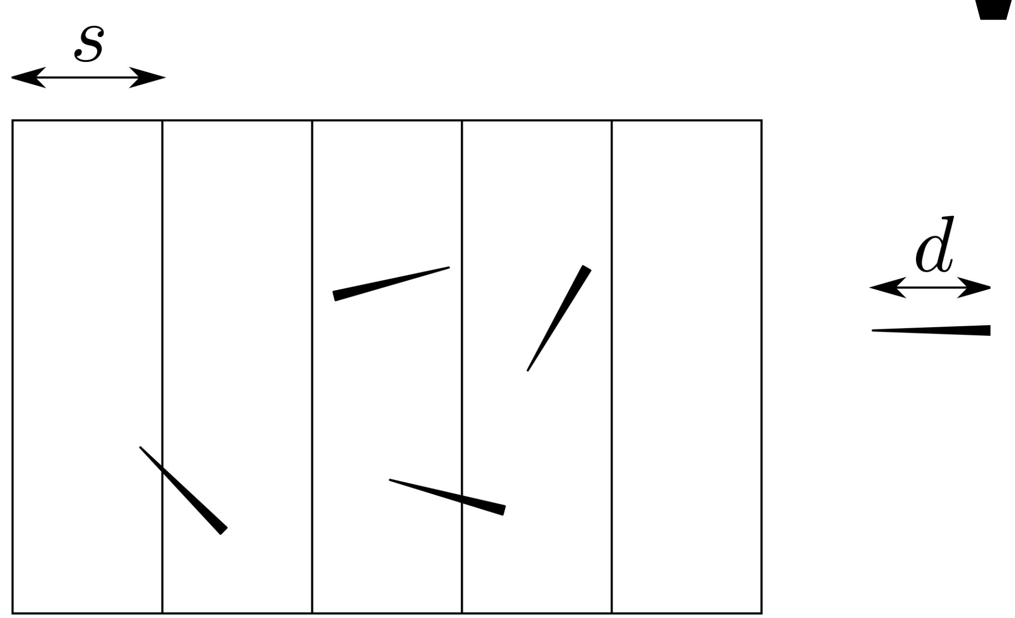

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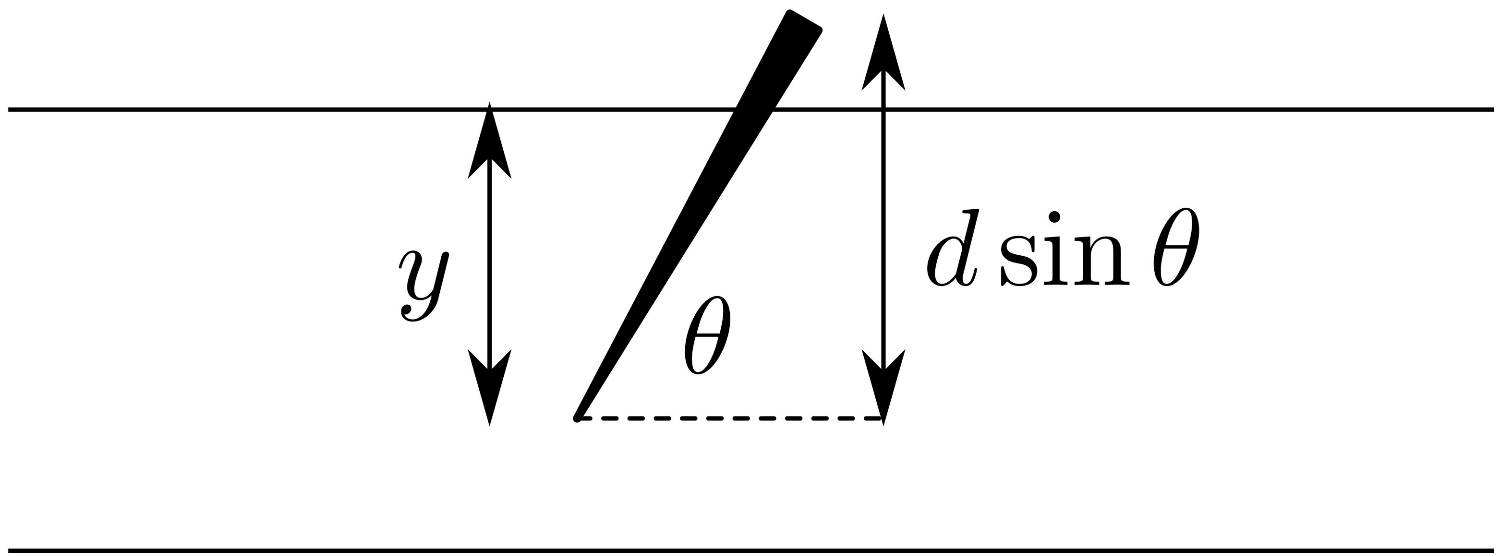

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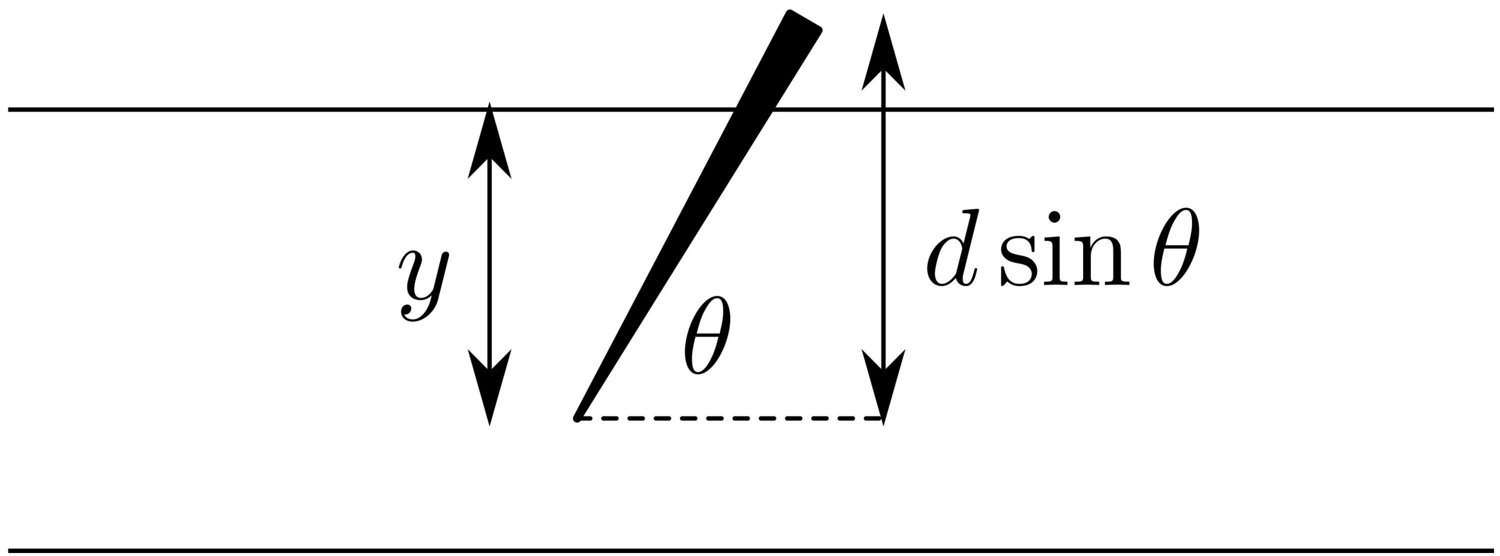

- y varies from 0 to *s*
- θ varies from 0 to 180 degrees (0 to π radians)
- The needle **crosses** if $y < d \sin \theta$

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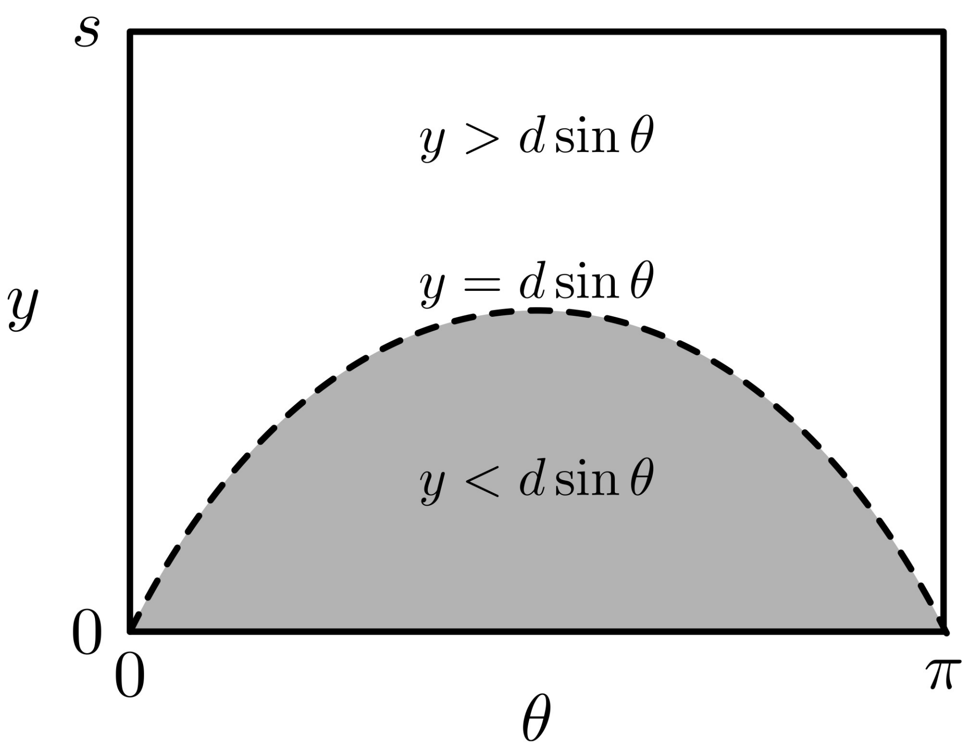

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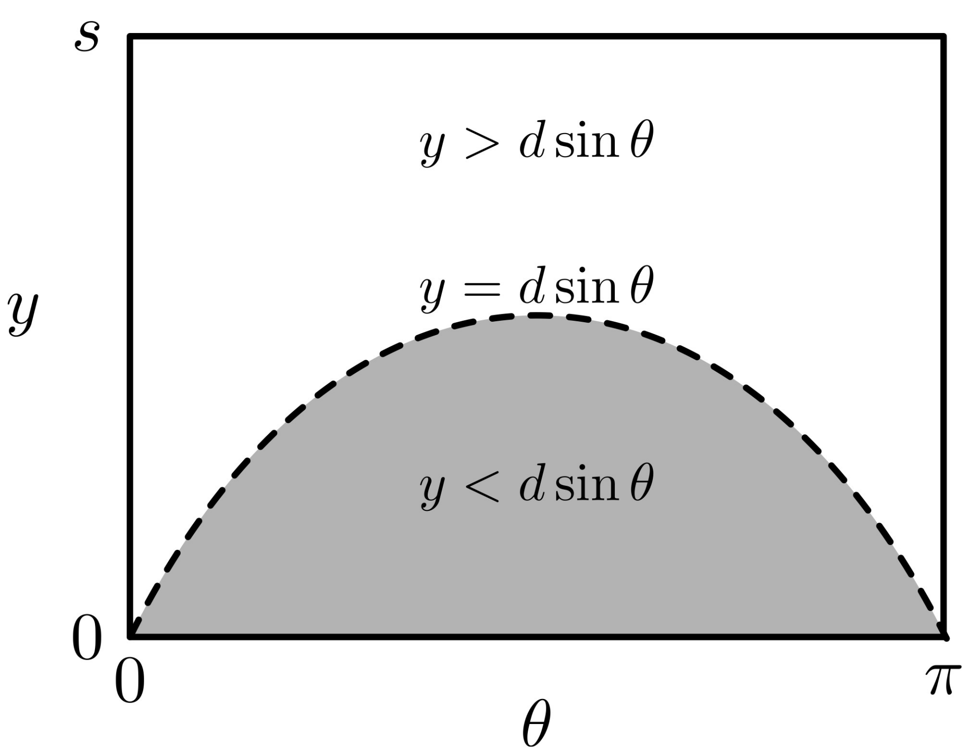

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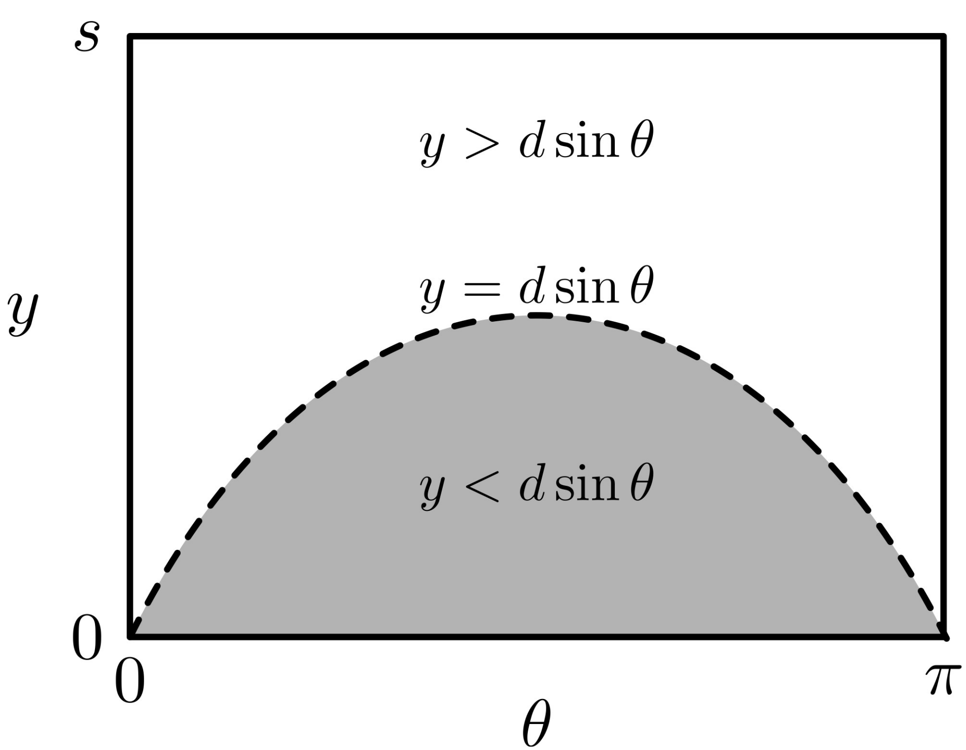

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Calculus!

□

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- But he does have a hardwood floor.

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- He has a needle.
- He doesn't know how long it is.
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- But he does have a hardwood floor.
- Throw it on the floor!

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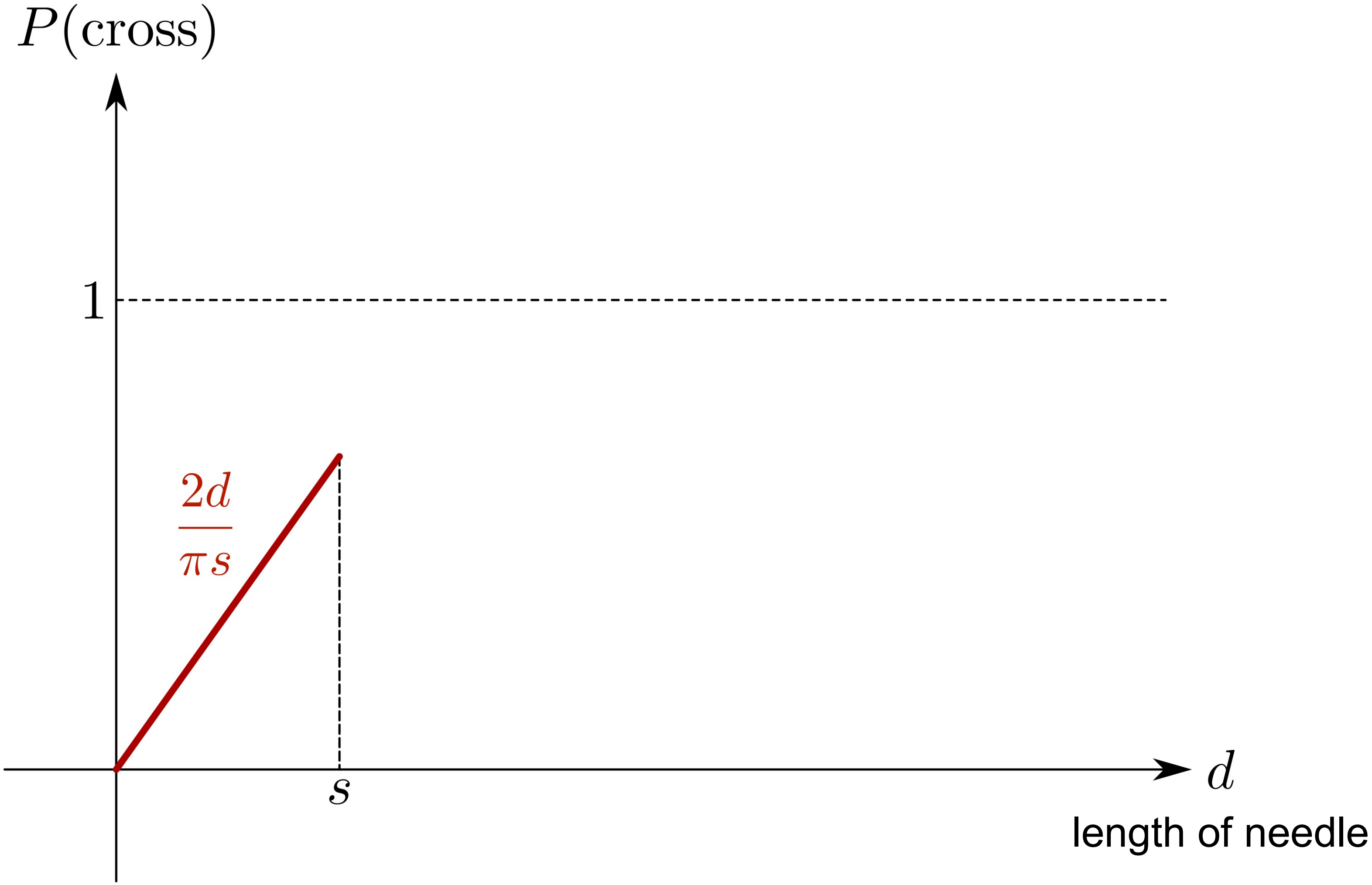

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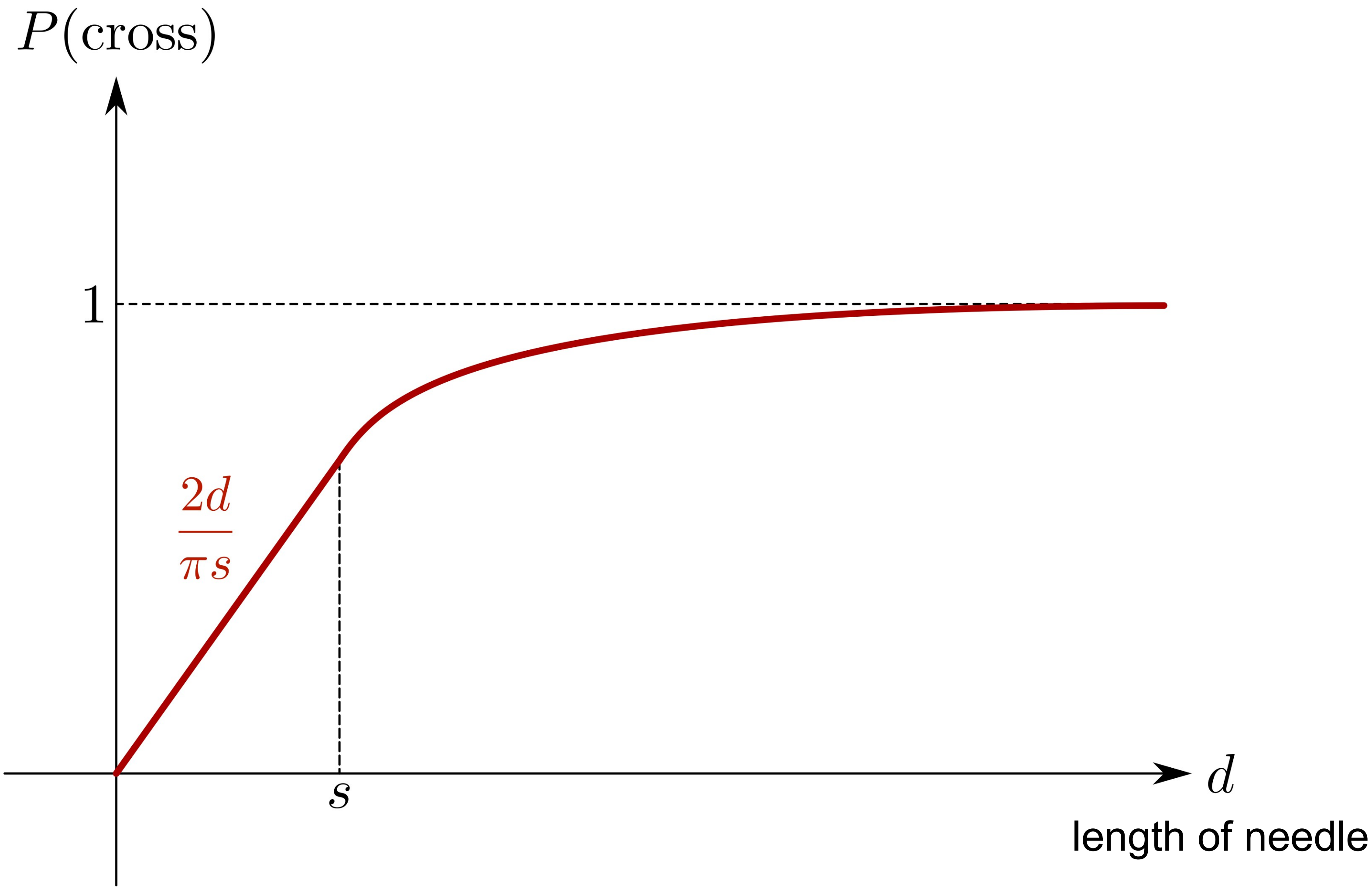

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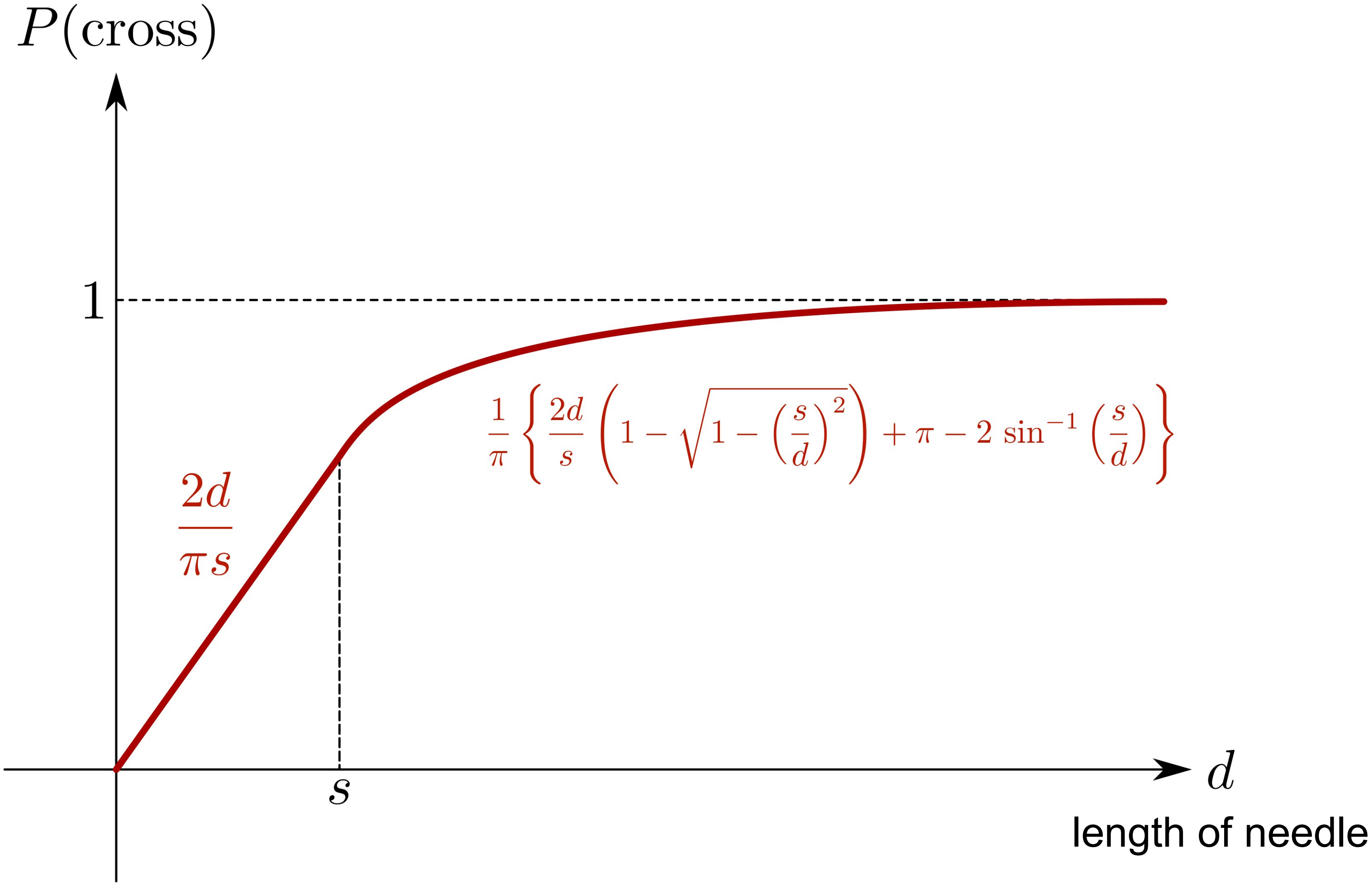

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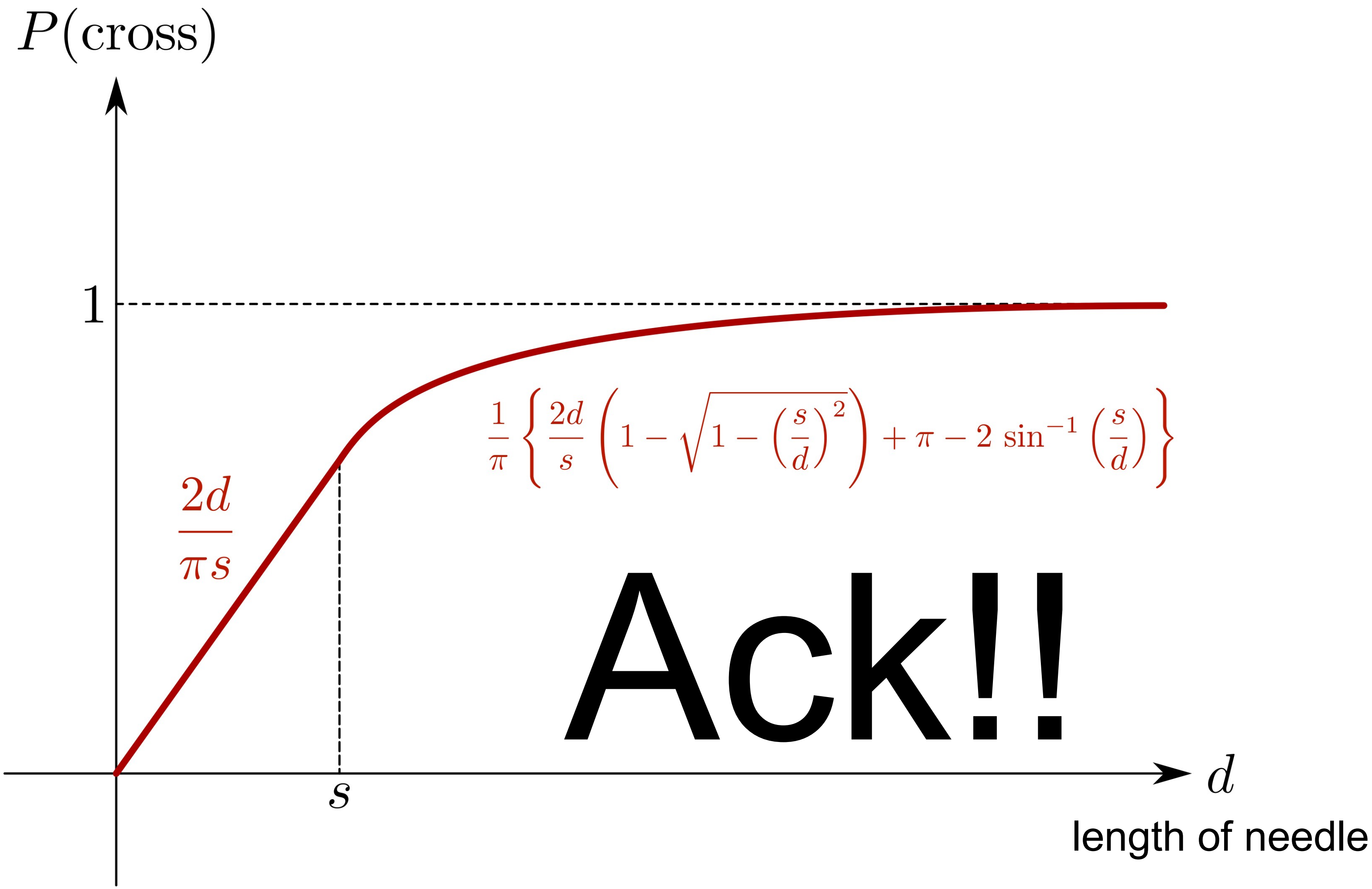

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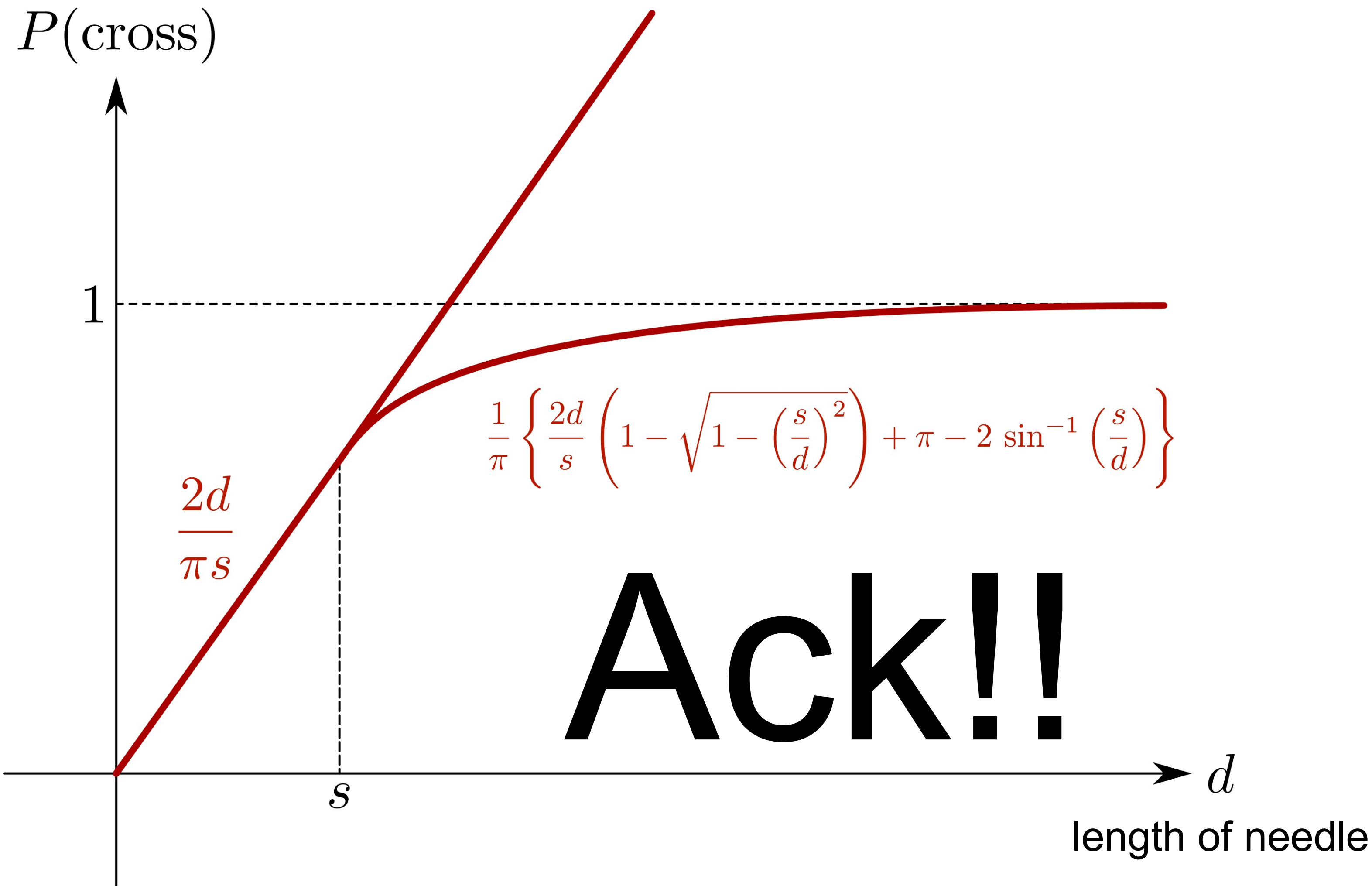

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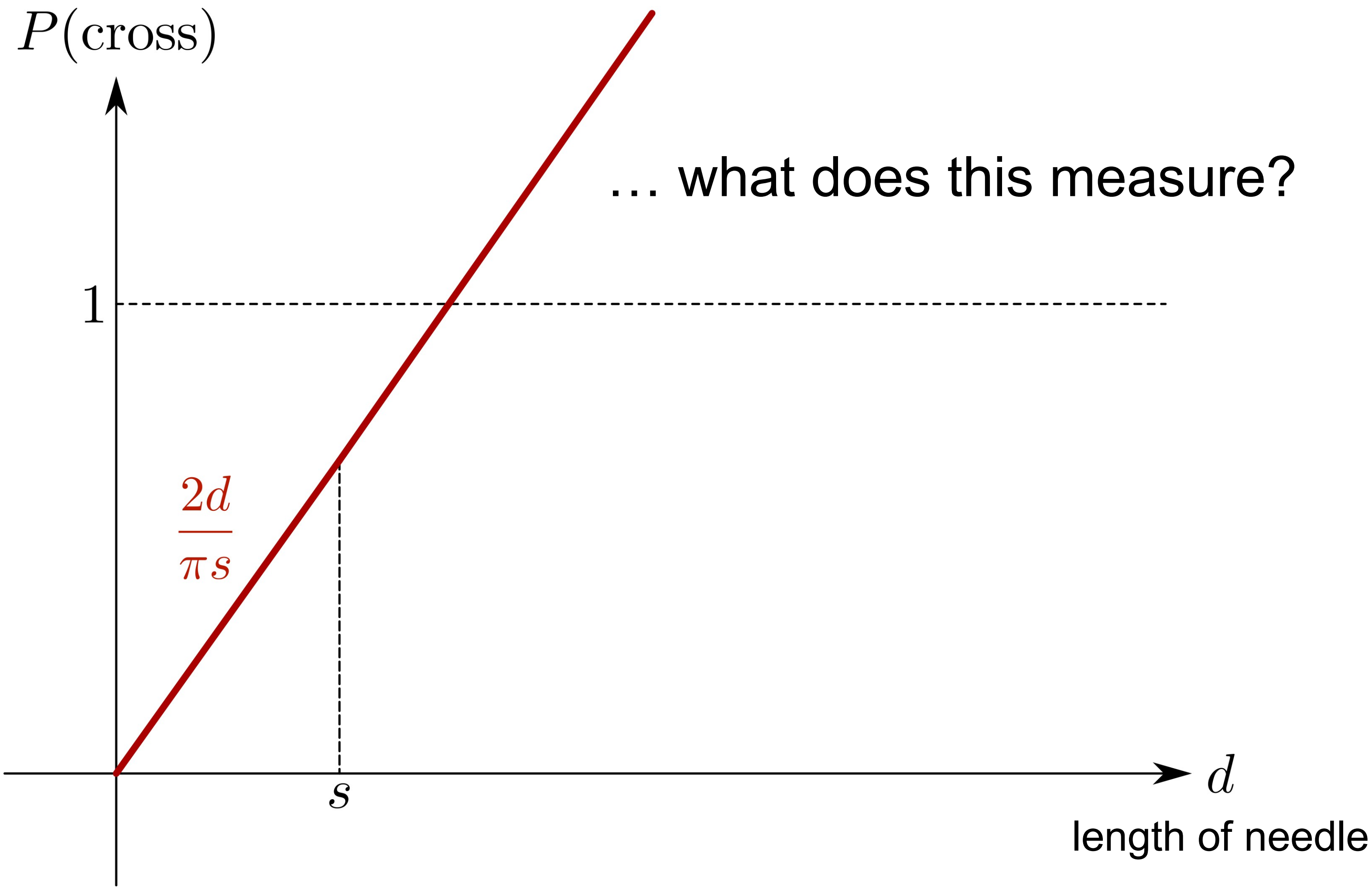

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(Barbier, 1860) Showed us the following:

- Throw a needle of **any** length *d* on hardwood floor of spacing *s*.

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(Barbier, 1860) Showed us the following:

- Throw a needle of **any** length *d* on hardwood floor of spacing *s*.

Then: $$\text{Average}(\#\text{ crossings}) = \frac{2d}{\pi s}$$

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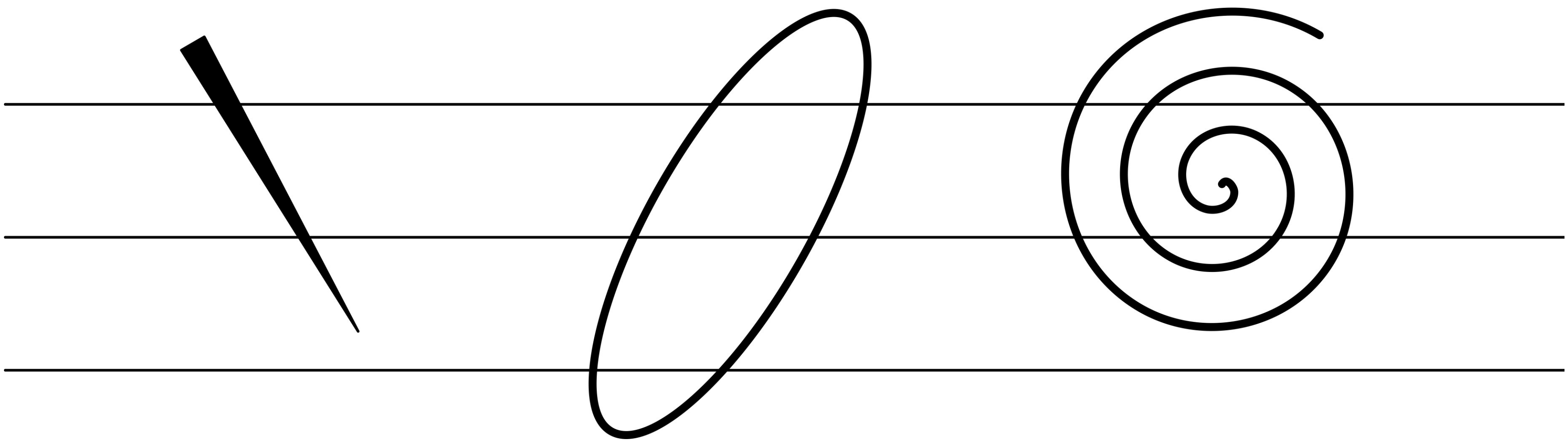
Barbier's formula works even for **noodles!**

- What is a noodle?
  e.g.



2 crossings   6 crossings   7 crossings

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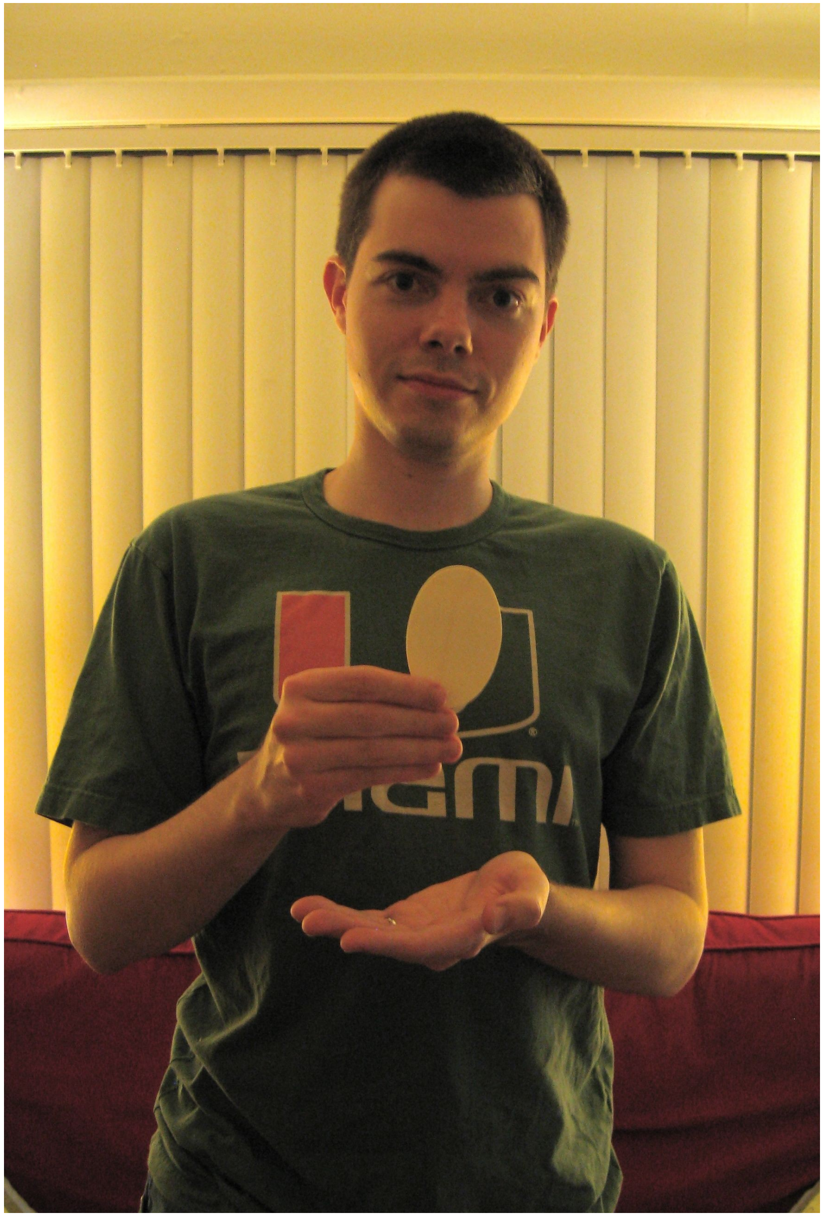
- He wants to know the perimeter.
- What should he do?



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The man is back.

- Now he has a shape.
- He wants to know the perimeter.
- What should he do?
- Throw it on the floor!
- Watch him go!

Observations

1 1 1 1 1 1 1 2 1 2 1 1 2 1 2 1

1 1 2 1 1 2 1 1 1 2 1 1 1 2 1

1 1 1 2 1 2 2 2 2 2 2 2 1 2 1 1

1 2 1 1 1 1 2 2 2 1 2 1 1 1

1 1 1 2 1 1 1 2 1 1 2 1 2 1 1

1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 1

64 "1"s
27 "2"s

Total # crossings $= 2(64\cdot 1 + 27\cdot 2) = 236$

Average # crossings $= \frac{236}{64+27} = \frac{236}{91}$

Barbier says $\frac{236}{91} \approx \frac{2d}{\pi s}$

or perimeter $d \approx \frac{236\,\pi s}{2\cdot 91}$

Know: spacing $s \approx 2.25$ inches

So . . .

perimeter $d \approx \frac{236(3.14)(2.25)}{2\cdot 91} =$ **9.16 inches**

perimeter of the shape

= approx. 9.16 inches

Thank you!

Thank you!

(wait for applause)