

Table A.17. The characters of the binary dihedral group,  $n$  even

Character	Conjugacy class $Cl(g)$ and its order under it			
	$Cl(1)$	$Cl(-1)$	$Cl(a^k)$ for $k = 1, \dots, n-1$	$Cl(b)$ $Cl(ba)$
$\chi_i$	1	1	2	$n$ $n$
$\chi_1$	1	1	1	1 1
$\chi_2$	1	1	1	-1 -1
$\chi_3$	1	-1	$(-1)^k$	$i$ $-i$
$\chi_4$	1	-1	$(-1)^k$	$-i$ $i$
$\chi'_1$	2	-2	$\xi^k + \xi^{-k}$	0 0
$\chi'_2$	2	2	$\xi^{2k} + \xi^{-2k}$	0 0
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$\chi'_{n-1}$	2	$(-2)^{n-1}$	$\xi^{k(n-1)} + \xi^{-k(n-1)}$	0 0

Table A.18. The characters of the binary dihedral group,  $n$  odd

Character	Conjugacy class $Cl(g)$ and its order under it			
	$Cl(1)$	$Cl(-1)$	$Cl(a^k)$ for $k = 1, \dots, n-1$	$Cl(b)$ $Cl(ba)$
$\chi_i$	1	1	2	$n$ $n$
$\chi_1$	1	1	1	1 1
$\chi_2$	1	1	1	-1 -1
$\chi_3$	1	-1	$(-1)^k$	1 -1
$\chi_4$	1	-1	$(-1)^k$	-1 1
$\chi'_1$	2	-2	$\xi^k + \xi^{-k}$	0 0
$\chi'_2$	2	2	$\xi^{2k} + \xi^{-2k}$	0 0
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$\chi'_{n-1}$	2	$(-2)^{n-1}$	$\xi^{k(n-1)} + \xi^{-k(n-1)}$	0 0

$$R^2 = S^n = T^2 = RST = -1, \tag{A.67}$$

(A.5) holds for  $(2, 2, n)$ . characters of the binary dihedral group  $\mathcal{D}$  are given in Table A.17 ( $n$  and Table A.18 ( $n$  is odd). In these tables  $\xi$  is a primitive  $2n$ -th root

### A.6.3 The binary icosahedral group

The binary icosahedral group  $\mathcal{I}$  has the presentation:

$$\mathcal{I} = \{a, b \mid a^5 = b^3 = (ba)^2 = -1\}. \tag{A.68}$$

Table A.19. The characters of the binary icosahedral group

Character	The conjugacy class $Cl(g)$ and its order (under it)									
	$Cl(1)$	$Cl(-1)$	$Cl(a)$	$Cl(a^2)$	$Cl(a^3)$	$Cl(a^4)$	$Cl(b)$	$Cl(b^2)$	$Cl(ab)$	$Cl(ab^2)$
$\chi_i$	1	1	12	12	12	12	20	20	30	30
$\chi_1$	1	1	1	1	1	1	1	1	1	1
$\chi_2$	2	-2	$\mu^+$	$-\mu^-$	$\mu^-$	$-\mu^+$	1	-1	0	0
$\chi_3$	2	-2	$\mu^-$	$-\mu^+$	$\mu^+$	$-\mu^-$	1	-1	0	0
$\chi_4$	3	3	$\mu^+$	$\mu^-$	$\mu^-$	$\mu^+$	0	0	-1	-1
$\chi_5$	3	3	$\mu^-$	$\mu^+$	$\mu^+$	$\mu^-$	0	0	-1	-1
$\chi_6$	4	-4	1	-1	1	-1	-1	1	0	0
$\chi_7$	4	4	-1	-1	-1	-1	1	1	0	0
$\chi_8$	5	5	0	0	0	0	0	-1	-1	1
$\chi_9$	6	-6	-1	1	-1	1	0	0	0	0

e. values  $2 \quad -2 \quad \mu^+ \quad -\mu^- \quad \mu^- \quad -\mu^+ \quad 1 \quad -1 \quad 1 \quad 0$

By setting

$$R := b, S := a, T := ba,$$

we deduce that presentation (A.68) is equivalent to the following presentation:

$$R^3 = S^5 = T^2 = RST = -1, \tag{A.69}$$

i.e., eq. (A.5) holds for  $(2, 3, 5)$ .

Let  $\mu^+, \mu^-$  be as follows:

$$\mu^+ = \frac{1}{2}(1 + \sqrt{5}), \text{ and } \mu^- = \frac{1}{2}(1 - \sqrt{5}).$$

The characters of the binary icosahedral group  $\mathcal{I}$  are given in Table A.19.