Math 461 F | **Spring 2011**
Homework 6 | **Drew Armstrong**

Reading.

Section 6.4, 6.5

Book Problems.

6.4: 1, 4, 8.

Additional Problems.

A.1. Euler showed that every real polynomial of the form $x^4 + \alpha x^2 + \beta x + \gamma$ factors into two real quadratics. Use his result to **prove** that every real polynomial of the form $ax^4 + bx^3 + cx^2 + dx + e$ factors into two real quadratics. (Hint: Use a change of variables to turn your polynomial into Euler's polynomial.)

A.2. Suppose that a given real quartic equation has roots a, b, c, d in some field $\mathbb{E} \supseteq \mathbb{R}$. (Today we know that these roots must be complex, but in times past their nature was mysterious.) Now let $f(a, b, c, d)$ be some function of a, b, c, d that is invariant under any permutation of the roots. Explain why $f(a, b, c, d)$ is a **real** number. (Hint: The Fundamental Theorem of Symmetric Functions.)

A.3. Let a, b, c, d be the roots of some real quartic equation with no x^3 term (i.e. we have $a + b + c + d = 0$.) Let $p = a + b$, $q = a + c$, and $r = a + d$, so that $-p = c + d$, $-q = b + d$, and $-r = b + c$. **Prove** that pqr is a real number, and hence $-p^2q^2r^2$ is a **negative** real number. (Hint: Show that pqr is invariant under permuting $a \leftrightarrow b$, or $a \leftrightarrow c$, or $a \leftrightarrow d$. Hence it's invariant under **any** permutation of a, b, c, d.)