Math 311 **Spring 2023**
Practice for Exam 1 **Drew Armstrong**

---

1. Slope Fields. Just sketch. Don't solve.

(a) Sketch the slope field of $dy/dx = y$ and the two solutions with $y(0) = 1, -1$.
(b) Sketch the slope field of $dy/dx = x$ and the two solutions with $y(0) = 1, -1$.
(c) Sketch the slope field of $dy/dx = 5 - y$ and the two solutions with $y(0) = 4, 6$.
(d) Sketch the slope field of $dy/dx = y(5 - y)$ and the three solutions with $y(0) = -1, 1, 6$.

2. Direct Integration. Solve the initial value problems.

(a) $x'' = 5$; $x(0) = 1$, $x'(0) = 3$
(b) $x'' = t$; $x(0) = 1$, $x'(0) = 3$
(c) $x' = \sin t$; $x(0) = 1$
(d) $x' = e^{3t}$; $x(0) = 4$

3. Free Fall (No Air Resistance). Suppose that the height of a projectile near the surface of a planet satisfies $h''(t) = -1$. Suppose the projectile is launched from a height $h(0) = 1$ with initial speed $h'(0) = 4$ (up). When does the projectile hit the ground ($h = 0$)?

4. Separation of Variables. Solve the initial value problems.

(a) $dy/dx = 2y - 3$; $y(0) = 1$
(b) $dy/dx = y^2$; $y(0) = 1$
(c) $dy/dx = xy$; $y(0) = 1$
(d) $dy/dx = x/y$; $y(0) = 1$
(e) $dy/dx = y(1 - y)$; $y(0) = 1/2$. [Hint: $\frac{1}{y(1-y)} = \frac{1}{y} + \frac{1}{1-y}$.]

5. Newton's Law of Cooling. Suppose the temperature of a cup of coffee satisfies $u'(t) = 5 - u(t)$ and $u(0) = 7$. Find a formula for $u(t)$ and compute the limit of $u(t)$ as $t \to +\infty$.

6. Free Fall (Air Resistance). Suppose the velocity of a projectile near the surface of a planet satisfies $dv/dt = -1 - v$. Suppose the projectile is dropped, so that $v(0) = 0$. Find a formula for $v(t)$ and compute the limit of $v(t)$ as $t \to +\infty$.

7. Integration Factors. Solve the initial value problems.

(a) $x' + x = 1$; $x(0) = 4$
(b) $x' + x = t$; $x(0) = 4$
(c) $x' = tx + t$; $x(0) = 4$

8. Trigonometry.

(a) Express $3\cos t + 4\sin t$ in the form $C\cos(t - \alpha)$.
(b) Express $(2 + i)e^{it} + (2 - i)e^{-it}$ in the form $A\cos t + B\sin t$.

9. Linear, Homogeneous, Constant Coefficients. Solve the initial value problems.

(a) $y'' - y = 0$; $y(0) = 1$; $y'(0) = 2$
(b) $y'' + 4y' + 4y = 0$; $y(0) = 1$; $y'(0) = 2$
(c) $y'' + 4y' + 5y = 0$; $y(0) = 1$; $y'(0) = 2$

10. Damped Oscillations. The equation $mx'' + \gamma x' + kx = 0$ represents a damped oscillator with mass $m > 0$, stiffness $k > 0$ and friction $\gamma > 0$. Assume that $\gamma^2 - 4mk < 0$ so the characteristic equation $m\lambda^2 + \gamma\lambda + k = 0$ has complex roots. In this case make a rough sketch of the solution with $x(0) = 1$ and $x'(0) = 1$. (You do not need to compute a formula.)