Math 309 **Summer B 2018**
Homework 2 **Drew Armstrong**

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1. De Morgan's Laws.

(a) Let $A, B \subseteq U$ be any subsets of the universal set. Use Venn diagrams to show that

$$(A \cup B)^c = A^c \cap B^c \qquad \text{and} \qquad (A \cap B)^c = A^c \cup B^c.$$

(b) Let P, Q be any logical statements. Use truth tables to show that

$$\neg(P \vee Q) = \neg P \wedge \neg Q \qquad \text{and} \qquad \neg(P \wedge Q) = \neg P \vee \neg Q.$$

2. The Contrapositive. Let P and Q be logical statements. We define the statement $P \Rightarrow Q$ (read as "P implies Q") by the formula

$$P \Rightarrow Q := (\neg P) \vee Q = (\text{NOT}\, P)\, \text{OR}\, Q.$$

(a) Draw the truth table for this function.
(b) Use a truth table or another method to show that "$P \Rightarrow Q$" is the same as "$\neg Q \Rightarrow \neg P$."
(c) If R is another logical statment, use Problem 1 and part (b) to show that

$$\text{“}P \Rightarrow (Q \vee R)\text{”} = \text{“}(\neg Q \wedge \neg R) \Rightarrow \neg P.\text{”}$$

3. Application. We way that an integer $n \in \mathbb{Z}$ is **odd** when $n = 2k + 1$ for some $k \in \mathbb{Z}$. We that n is **even** when it is not odd. Consider any two integers $m, n \in \mathbb{Z}$ and use Problem 2(c) to prove the following statement:

"If mn is even then m is even or n is even."

[Hint: Let P = "mn is even," Q = "m is even," and R = "n is even."]

4. Counting Functions. Let S be a finite set with $\#S$ elements and let T be a finite set with $\#T$ elements.

(a) Write a formula for the number of elements of $S \times T$, i.e., the number of ordered pairs (s, t) with $s \in S$ and $t \in T$.
(b) Write a formula for the number of functions from S to T.
(c) Use your answers from parts (a) and (b) to compute the number of Boolean functions with 2 inputs and 1 output. [Hint: By definition these are the functions from $\{T, F\} \times \{T, F\}$ to $\{T, F\}$.]

5. Counting Subsets.

(a) Explicitly write down all of the subsets of $\{1, 2, 3\}$.
(b) Explicitly write down all of the functions $\{1, 2, 3\} \to \{T, F\}$.
(c) If S is any set, let $\text{Sub}(S)$ be the set of subsets of S and let $\text{Fun}(S, \{T, F\})$ be the set of functions from S to $\{T, F\}$. Find a $1 : 1$ correspondence between these two sets:

$$\text{Sub}(S) \leftrightarrow \text{Fun}(S, \{T, F\}).$$

(d) If S is a finite set with $\#S$ elements, use Problem 4(b) to conclude that S has $2^{\#S}$ different subsets.