Math 309 **Summer 2014**
Quiz 2 Solutions **Drew Armstrong**

Throughout this quiz, P and Q are Boolean variables.

1. Write out the truth table for $P \vee Q$.

Proof.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

□

2. Write out the truth table for $P \wedge Q$.

Proof.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

□

Now consider the Boolean function $\varphi : \{T, F\}^2 \to \{T, F\}$ defined by the following table:

P	Q	$\varphi(P,Q)$
T	T	T
T	F	F
F	T	T
F	F	T

3. Write out the truth table for $\neg\,\varphi(P,Q)$.

Proof.

P	Q	$\neg\,\varphi(P,Q)$
T	T	F
T	F	T
F	T	F
F	F	F

□

4. Tell me a **formula** for $\neg\,\varphi(P,Q)$ in terms of $\vee, \wedge, \neg$. [Hint: Disjunctive normal form.]

Proof. The only T is in the (T, F) row of the truth table. This row corresponds to the formula $P \wedge \neg Q$ (and we can think of it as a region of the corresponding Venn diagram). Therefore the disjunctive normal form is

$$\neg\,\varphi(P,Q) = P \wedge \neg Q.$$

□

5. Now tell me a **formula** for $\varphi(P,Q)$. [Hint: Use Problem 4 and de Morgan's identity.]

Proof. We can read the disjunctive normal form of $\varphi(P,Q)$ from the truth table above. There are three T's in the rows (T,T), (F,T) and (F,F). Therefore the disjunctive normal form is

$$\varphi(P,Q) = (P \wedge Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q).$$

However, this doesn't look very nice. We get a more compact formula by using Problem 4 and applying de Morgan's identity. We have

$$\begin{aligned} \neg\,\varphi(P,Q) &= P \wedge \neg Q \\ \varphi(P,Q) &= \neg(P \wedge \neg Q) \\ \varphi(P,Q) &= \neg P \vee \neg\neg Q \\ \varphi(P,Q) &= \neg P \vee Q. \end{aligned}$$

□

[Remark: The Boolean function φ is very important in mathematics. In this class we will denote it by "$P \Rightarrow Q$" $:= \varphi(P,Q)$ and we will read the statement "$P \Rightarrow Q$" as "if P then Q", or "P implies Q". (WARNING: This definition might not **totally** agree with your intuition about the words "if ... then ...". For example, do you regard "if $1+1=3$ then $1+1=5$" to be a true statement? I do.) Mathematical proofs and logical arguments are built from these arrows.]