Math 309 **Fall 2014**
Homework 5 **Drew Armstrong**

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1. De Morgan's Law. Let U be a set and consider the following logical statement depending on an integer n. We will call this statement $P(n)$:

"For **any** n sets $A_1, A_2, \ldots, A_n \subseteq U$ we have $(A_1 \cup A_2 \cup \cdots A_n)^c = A_1^c \cap A_2^c \cap \cdots \cap A_n^c$."

(a) Explain why $P(2)$ is a true statement.
(b) Fix $n \geq 2$ and **assume** for induction that $P(n)$ is a true statement. In this hypothetical case, show that the statement $P(n+1)$ is also true. [Hint: We proved something very similar in class.]
(c) What do you conclude?

2. Two Biased Dice. Suppose you have two 4-sided dice, one red and one blue. Suppose that each of these dice has probability distribution:

k	1	2	3	4
$P(k)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{8}$

You roll the two dice and record the outcome.
(a) What is the sample space of this experiment?
(b) Compute the probability of each possible outcome. [Hint: Multiply.] Verify that the sum of the probabilities equals 1.
(c) What is the probability that "the sum of the dice is 6"?
(d) What is the probability that "the sum of the dice is 6 **or** the red die shows 3"?

3. The Birthday Problem. Suppose there are n people in a room and you record all of their birthdays as a number between 1 and 365 (assume no one was born on February 29).
(a) What is the sample space? How many elements does it have?
(b) Show that the number of outcomes in which no two people have the same birthday is

$$365 \cdot 364 \cdot 363 \cdots (365 - n + 1).$$

(c) Now let's assume that each of the 365 days is equally likely to be someone's birthday. In this case, what is the probability that **no two people have the same birthday**?
(d) Following from part (c), what is the probability that **there exist two people in the room with the same birthday**? Use a computer to find the smallest n such that this probability is greater than $1/2$.

4. Yahtzee. Suppose you roll 5 **fair** 6-sided dice.
(a) What is the sample space?
(b) Since the dice are fair, each outcome has the same probability. What is this probability?
(c) When all 6 dice show the same number, this is called "rolling a Yahtzee". What is the probability of rolling a Yahtzee? [Hint: How many ways can it happen?]
(d) Suppose you roll the dice n times. What is the probability that you will roll a Yahtzee **exactly k times**? [Hint: Each roll is equivalent to a biased coin flip.]
(e) Suppose you roll the dice 1000 times. What is the probability that you will roll a Yahtzee **at least once**?