Math 309 **Fall 2014**
Homework 1 **Drew Armstrong**

For all positive integers p and n we consider the sum of the first n p-th powers:

$$S_p(n) := 1^p + 2^p + 3^p + \cdots + n^p = \sum_{k=1}^{n} k^p.$$

We have already seen that $S_1(n) = n(n+1)/2$ and $S_2(n) = n(n+1)(2n+1)/6$. Now you will show that $S_3(n) = n^2(n+1)^2/4$ using the technique of induction.

1. Verify that $S_3(n) = n^2(n+1)^2/4$ is true for small values of n.

2. Let n be some "fixed, but arbitrary" positive integer. Show that **if** $S_3(n) = n^2(n+1)^2/4$ is true **then** $S_3(n+1) = (n+1)^2(n+2)^2/4$ is also true. [Hint: Take out common factors whenever you can.]

3. Use your knowledge of $S_1(n)$, $S_2(n)$, and $S_3(n)$ to find a closed form for the following sum:

$$\sum_{k=1}^{n} k(k+1)(k+2).$$

For the next two problems, consider the recurrence relation:

$$\boxed{F_n = F_{n-1} + 2n.}$$

4. (a) Compile a table of F_n with initial condition $F_0 = 0$.
(b) Compile a table of F_n with initial condition $F_2 = 5$.
(c) If $F_7 = x$ then what is F_3?

5. Find a **closed formula** for F_n with initial condition $F_0 = 0$. [Hint: Expand the recurrence to show that $F_n = 0 + 2 + 4 + \cdots + 2n$. Now what?]