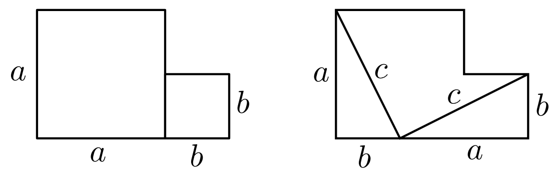
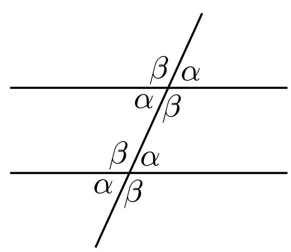
Math 230 D **Fall 2015**
Homework 1 **Drew Armstrong**

Problem 1. In this problem you will prove the Pythagorean Theorem by cutting up the same shape in two different ways.



[Hint: The three pieces of the right diagram can be rearranged into a square. Your proof should begin like this: "Consider a right angled triangle with side lengths a, b, and c, where c is is the hypotenuse. (Now draw a picture of the triangle.) In this case we will prove that $a^2 + b^2 = c^2$. To do this we will cut up the same shape in two different ways. Consider the following two diagrams..."]

Problem 2. Prove that the interior angles of any triangle sum to 180°. You may use the following two facts without proof. **Prop I.31:** Given a line ℓ and a point p not on ℓ, **it is possible** to draw a line through p parallel to ℓ. **Prop I.29:** If a line falls on two parallel lines, then the corresponding angles are equal, as in the following figure.



Problem 3. Look up Euclid's Proposition I.48. Tell me the statement and the proof. Your goal is to make everything as understandable as possible, especially to yourself but also to me. Imagine that you are trying to teach this to someone.

Problem 4. The dot product of the vectors $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$ is defined by $\mathbf{u} \bullet \mathbf{v} := u_1 v_1 + u_2 v_2$. The length $\|\mathbf{u}\|$ of a vector $\mathbf{u}$ satisfies $\|\mathbf{u}\|^2 = \mathbf{u} \bullet \mathbf{u}$.

(a) The vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{u} - \mathbf{v}$ form the three sides of a triangle. Draw this triangle.

(b) Use algebra (not geometry) to prove that $\|\mathbf{u} - \mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2(\mathbf{u} \bullet \mathbf{v})$.

(c) Use the formula from part (b) to prove the following statement:

"the vectors $\mathbf{u}$ and $\mathbf{v}$ are perpendicular if and only if $\mathbf{u} \bullet \mathbf{v} = 0$."

[Hint: Remember your picture from part (a). What do the Pythagorean Theorem and its converse tell you about this picture?]