Euclid wrote *The Elements* around 300BC. This was the definitive statement of the deductive method, which uses logic to rigorously derive (prove) theorems from self-evident first principles (axioms). What was self-evident to Euclid? His work is based on 10 axioms broken into two categories: 5 "postulates" and 5 "common notions". The postulates describe the contruction of geometric figures.

Euclid's "Postulates"

Let the following be postulated:
(P1) To draw a straight line from any point to any point.
(P2) To produce a finite straight line continuously in a straight line.
(P3) To describe a circle with any center and radius.
(P4) That all right angles are equal to one another.
(P5) That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles.

Does the fifth postulate seem self-evident to you? Euclid also described what kinds of logical deduction should be allowed. He based his logic on the following five rules of comparison.

Euclid's "Common Notions"

(CN1) Things which are equal to the same thing are equal to one another.
(CN2) If equals be added to equals, the wholes are equal.
(CN3) If equals be subtracted from equals, the remainders are equal.
(CN4) Things which coincide with one another are equal to one another.
(CN5) The whole is greater than the part.

In the 13 volumes of the *Elements*, Euclid used these basic principles to deduce (prove) all of Classical Greek mathematics. The climax of Book I is the Pythagorean Theorem, which is Euclid's 47th proposition (theorem).

Q: What is so "self-evident" about Euclid's axioms?

A: They are based on visual/spacial intuition; they model the process of using a straightedge and compass to draw precise figures.

Euclid's *Elements* was the standard for mathematical rigor until the late 19th century (amazing!!). At that point, mathematicians realized that some of Euclid's physical intuition was not really self-evident. (Einstein would soon show that gravity comes from a curved *non-Euclidean* geometry.) In 1899, David Hilbert filled in the details and brought Euclid's Elements to a modern standard of rigor. Hilbert's modern version uses **20 axioms!**