Math 224 **Summer 2019**
Quiz 3 **Drew Armstrong**

Problem 1. Recall that a standard deck of 52 cards contains 13 card of each suit (clubs, diamonds, hearts and spades). Suppose that two cards are dealt without replacement from a standard deck and consider the events

$$A = \{\text{first card is a club}\},$$
$$B = \{\text{second card is a club}\}.$$

(a) Compute $P(A)$ and $P(B)$.

Since there are 13 clubs in 52 equally likely cards, we have $P(A) = 13/52 = 1/4$. By ignoring the first card, I claim that we also have $P(B) = 1/4$. If you don't believe this, see below.

(b) Compute the forwards probabilities $P(B|A)$ and $P(B|A')$.

If the first card is a club then there are 12 clubs in the remaining 51 cards:

$$P(B|A) = \frac{12}{51}.$$

If the first card is not a club then there are 13 clubs in the remaining 51 cards:

$$P(B|A') = \frac{13}{51}.$$

This allows us to verify our computation of $P(B)$:

$$\begin{aligned} P(B) &= P(A \cap B) + P(A' \cap B) \\ &= P(A)P(B|A) + P(A')P(B|A') \\ &= \frac{1}{4} \cdot \frac{12}{51} + \frac{3}{4} \cdot \frac{13}{51} \\ &= \frac{1}{4}\left(\frac{12 + 3 \cdot 13}{51}\right) = \frac{1}{4} \cdot \frac{51}{51} = \frac{1}{4}. \end{aligned}$$

(c) Compute the backward probability $P(A|B)$.

By definition of conditional probability we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B|A)}{P(B)} = \frac{(1/4)(12/51)}{1/4} = \frac{12}{51}.$$

I guess that makes sense. It's like the order of the two cards doesn't matter.

Problem 2. An urn contains 2 red and 2 green balls. Suppose you grab 2 balls without replacement and let R be the number of red balls you get.

(a) Compute the probability mass function $P(R = k)$.

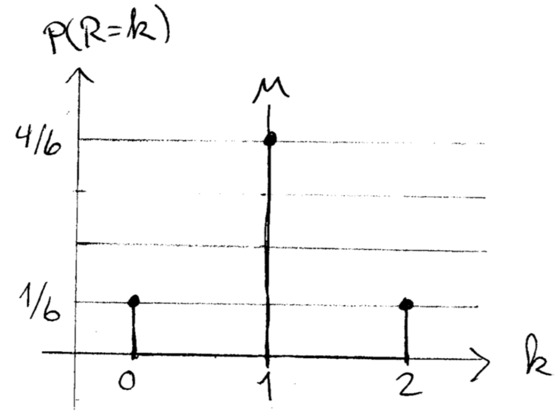
This is a "hypergeometric random variable" with pmf given by

$$P(R = k) = \frac{\binom{2}{k}\binom{2}{2-k}}{\binom{4}{2}}$$

Here is a table:

k	0	1	2
$P(R = k)$	$\frac{\binom{2}{0}\binom{2}{2}}{\binom{4}{2}} = \frac{1}{6}$	$\frac{\binom{2}{1}\binom{2}{1}}{\binom{4}{2}} = \frac{4}{6}$	$\frac{\binom{2}{2}\binom{2}{0}}{\binom{4}{2}} = \frac{1}{6}$

(b) Draw a picture of the probability mass function from part (a).



(c) Compute the expected value $E[R]$ and add this to your picture.

The center of mass (expected value) is

$$\begin{aligned}\mu = E[R] &= 0 \cdot P(R = 0) + 1 \cdot P(R = 1) + 2 \cdot P(R = 2) \\ &= 0 \cdot \frac{1}{6} + 1 \cdot \frac{4}{6} + 2 \cdot \frac{1}{6} = 1.\end{aligned}$$

We could have guessed this because the distribution is symmetric.