Math 224 **Summer 2019**
Homework 5 **Drew Armstrong**

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1. Let $X_1, X_2, \ldots, X_{15}$ be independent and identically distributed (iid) random variables. Suppose that each X_i has pdf defined by the following function:

$$f(x) = \begin{cases} \frac{3}{2} \cdot x^2 & \text{if } -1 \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Compute $E[X_i]$ and $\text{Var}(X_i)$.
(b) Consider the sum $\Sigma X = X_1 + X_2 + \cdots + X_{15}$. Compute $E[\Sigma X]$ and $\text{Var}(\Sigma X)$.
(c) The Central Limit Theorem says that ΣX is approximately normal. Use this fact to estimate the probability $P(-0.18 \le \Sigma X \le 0.36)$.

2. Suppose that $n = 48$ seeds are planted and suppose that each seed has a probability $p = 2/3$ of germinating. Let X be the number of seeds that germinate and use the Central Limit Theorem to estimate the probability $P(30 \le X \le 33)$ that between 30 and 33 seeds germinate (inclusive). Don't forget to use a continuity correction.

3. Suppose that a coin is flipped 100 times and let X be the number of heads. We will use $\hat{p} = X/100$ as an (unbiased) estimator for the unknown probability of heads: $p = P(H)$.

(1) Assuming that the coin is **fair** ($p = 1/2$), compute $E[\hat{p}]$ and $\text{Var}(\hat{p})$.
(2) In order to test the hypothesis $H_0 =$ "$p = 1/2$" against $H_1 =$ "$p > 1/2$" we flip the coin 100 times and get heads 60 times. Should you reject H_0 in favor of H_1 at the 5% level of significance? At the 2.5% level of significance? At the 1% level of significance? [Hint: $\hat{p}$ is approximately normal.]

4. Let p be the true proportion of voters in a population who intend to vote for candidate A. Suppose that we polled 1000 voters and 522 told us that they intend to vote for A. Use this to compute confidence intervals for p at the 90%, 95% and 99% confidence levels. [Hint: You may assume that the voters' responses are iid coin flips.]

5. Let $X_1, \ldots, X_{10}$ be an iid sample from a **normal distribution** with variance $\sigma^2 = 36$ and unknown mean μ. Suppose that the sample mean is measured to be

$$\overline{X} = \frac{1}{10}(X_1 + \cdots + X_{10}) = 50.$$

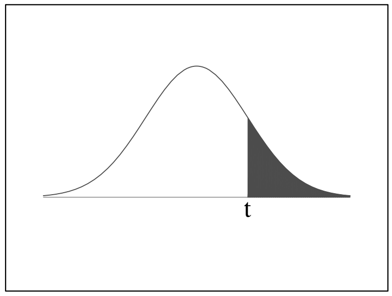
Use this to compute confidence intervals for μ at the 90%, 95% and 99% confidence levels. [Hint: $(\overline{X} - \mu)/\sigma$ has an exactly standard normal distribution.]

6. In order to estimate the average weight μ of a chocolate bar, a random sample of $n = 9$ bars from a production line are weighed, yielding the following results in grams:

21.40	18.85	18.55	19.40	19.15	22.45	22.80	22.20	23.15

Use this data to compute a 98% confidence interval for the average weight μ. [Hint: Assume that the weight of each chocolate bar is normally distributed and let $\overline{X}, S$ be the sample mean and standard deviation. Since $n = 9$ is a small number you should use the fact that $(\overline{X} - \mu)/\sqrt{S^2/n}$ has an approximate t-distribution with 8 degrees of freedom.]

t-Distribution Table



The shaded area is equal to α for $t = t_{\alpha}$.

df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
32	1.309	1.694	2.037	2.449	2.738
34	1.307	1.691	2.032	2.441	2.728
36	1.306	1.688	2.028	2.434	2.719
38	1.304	1.686	2.024	2.429	2.712
∞	1.282	1.645	1.960	2.326	2.576