Math 224 **Summer 2019**
Homework 2 **Drew Armstrong**

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1. Suppose that a fair s-sided die is rolled n times.

(a) If the i-th side is labeled a_i then we can think of the sample space S as the set of all words of length n from the alphabet $\{a_1, \ldots, a_s\}$. Find $\#S$.

(b) Let E be the event that "the 1st side shows up k_1 times, and ... and the s-th side shows up k_s times. Find $\#E$. [Hint: The elements of E are words of length n in which the letter a_i appears k_i times.]

(c) Compute the probability $P(E)$. [Hint: Since the die is fair you can assume that the outcomes in S are equally likely.]

2. Suppose that a fair six-sided die has 1 side painted red, 2 sides painted green and 3 sides painted blue. Suppose that you roll the die $n = 4$ times and let R, G, B be the number of times you see red, green and blue, respectively.

(1) Compute $P(R = 1, G = 1, B = 2)$.
(2) Compute $P(R = 1, G = 2, B = 1)$.
(3) Compute $P(R \geq 1)$. [Hint: Treat the die as a coin.]
(4) Compute $P(G = B)$. [Hint: What are the possible values of R, G, B?]

3. In a certain state lottery four numbers are drawn (one and at a time and with replacement) from the set $\{1, 2, 3, 4, 5, 6\}$. You win if any permutation of your selected numbers is drawn. Rank the following selections in order of how likely each is to win.

(a) You select $1, 2, 3, 4$.
(b) You select $1, 2, 3, 3$.
(c) You select $1, 1, 2, 2$.
(d) You select $1, 2, 2, 2$.
(e) You select $1, 1, 1, 1$.

4. A bridge hand consists of 13 (unordered) cards taken (at random and without replacement) from a standard deck of 52. Recall that a standard deck contains 13 hearts and 13 diamonds (which are red cards), 13 clubs and 13 spades (which ard black cards). Find the probabilities of the following hands.

(a) 2 hearts, 3 diamonds, 4 spades and 4 clubs.
(b) 2 hearts, 3 diamonds and 8 black cards.
(c) 5 red cards and 8 black cards.

5. Let $E, F \subseteq S$ be two events in a sample space and define the *conditional probability*:

$$P(E|F) = P(E \cap F)/P(F).$$

We interpret the number $P(E|F)$ as "the probability that E happens, assuming that F happens." Now suppose that two cards are drawn (in order and without replacement) from a standard deck of 52 and consider the events

$$A = \{\text{the first card is red}\}$$
$$B = \{\text{the second card is a diamond}\}.$$

In this case, compute the following probabilities:

$$P(A), \quad P(B), \quad P(B|A), \quad P(A \cap B), \quad P(A|B).$$

6. Consider a classroom containing n students. Suppose we record each student's birthday as a number between 1 and 365 (we ignore leap years). Let S be the sample space.

(a) Explain why $\#S = 365^n$.

(b) Let E be the event that {no two students have the same birthday}. Compute $\#E$.

(c) Assuming that all birthdays are equally likely, compute the probability of the event

$$E' = \{\text{at least two students have the same birthday}\}.$$

(d) Find the smallest value of n such that $P(E') > 50\%$.

7. It was not easy to find a formula for the entries of Pascal's Triangle. However, once we've found the formula it is not difficult to check that the formula is correct.

(a) Explain why $n! = n \cdot (n-1)!$.

(b) Use part (a) to verify that

$$\frac{(n-1)!}{(k-1)!(n-k)!} + \frac{(n-1)!}{k!(n-1-k)!} = \frac{n!}{k!(n-k)!}.$$

[Hint: Try to get a common denominator.]