Math 224 **Spring 2022**
Homework 1 **Drew Armstrong**

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1. A Fair Coin.

(a) Draw Pascal's triangle down to the sixth row. (The first row consists of a single 1.)
(b) Suppose a **fair** coin is flipped 5 times and let X be the number of heads you get. Use part (a) to compute the probabilities $P(X = k)$ for $k = 0, 1, 2, 3, 4, 5$.
(c) Use your answer from part (b) to compute the following probabilities:

$$P(X \geq 4), \quad P(X \text{ is odd}), \quad P(X \geq 1).$$

2. A Biased Coin. Consider a general coin with $P(H) = p$ and $P(T) = q$, where p and q are any real numbers satisfying $p, q \geq 0$ and $p + q = 1$.

(a) Suppose you flip the coin 5 times and let X be the number of heads you get. Use Problem 1(a) to compute the probabilities $P(X = k)$ for $k = 0, 1, 2, 3, 4, 5$. [Hint: Your answers will involve the unknown constants p and q.]
(b) Compute the probability $P(X \geq 1)$. [Hint: It is easier to compute $P(X = 0)$ and then use the fact that $P(X = 0) + P(X \geq 1) = 1$. Your answer will contain p and/or q.]

3. Working With Events. Suppose that you flip a **fair** coin 4 times. We we record each possible outcome as a sequence of H's and T's, such as "$THHT$".

(a) List the elements of the sample space S. What is $\#S$? Since the coin is fair, we can assume that each of these outcomes is equally likely, so the probably of any event $E \subseteq S$ is given by $P(E) = \#E/\#S$.
(b) List the elements in each of the following events:

$$A = \{\text{exactly 2 heads}\},$$
$$B = \{\text{an even number of heads}\},$$
$$C = \{\text{heads on the first flip, anything on the other flips}\}.$$

(c) Use parts (a) and (b) to compute the following probabilities:

$$P(A), \quad P(B), \quad P(C), \quad P(A \cap C), \quad P(A \cup B).$$

[Hint: The *intersection* $A \cap C$ is the set of outcomes that are in A **and** in C. The *union* $A \cup B$ is the set of outcomes that are in A **or** in B, or in both.]

4. Rolling a Pair of Fair Dice. Suppose that you roll a pair of **fair** six-sided dice, each with sides labeled $\{1, 2, 3, 4, 5, 6\}$. We will record the outcome that "the first die shows i and the second die shows j" with the symbol "ij".

(a) List the elements of the sample space S. What is $\#S$? Since the dice are fair, we can assume that each of these outcomes is equally likely, so the probability of any event $E \subseteq S$ is $\#E/\#S$.
(b) Compute the probability of getting a "double six", i.e., a 6 on both dice.
(c) Let X be the sum of the two numbers that show up on the dice. Use part (a) to compute the probabilities $P(X = k)$ for $X = 2, 3, 4, \ldots, 12$. [Hint: Count the number of outcomes corresponding to each value of X.]

5. The Chevalier's Problems.

(a) Consider a general coin with $P(H) = p$ and $P(T) = q$. Suppose you flip the coin n times and let X be the number of heads you get. Compute $P(X \geq 1)$. [Hint: Compare with Problem 2(b). Your answer may contain the letters n, p, q.]

(b) Suppose that you roll a fair six-sided die 4 times and let X be the number of "sixes" that you get. Compute $P(X \geq 1)$. [Hint: Think of a die roll as a "strange coin flip" with $H =$"you get a six" and $T =$"you don't get six". What are p and q in this case?]

(c) Suppose that you roll a pair fair six-sided dice 24 times and let X be the number of "double sixes" that you get. Compute $P(X \geq 1)$. [Hint: Think of one roll of the dice as a "very strange coin flip" with $H =$"you get a double six" and $T =$"you don't get double six". What are p and q in this case?]