1. Consider the scalar field

$$f(x,y,z) = xye^{z}.$$

a) Compute the gradient vector:

$$\nabla f = \left\langle \frac{df}{dx}, \frac{df}{dy}, \frac{df}{dz} \right\rangle$$

$$= \langle ye^{z}, xe^{z}, xye^{z} \rangle$$

b) Compute the equation of the tangent plane to the surface $f(x,y,z) = 2$ at the point $(2,1,0)$.

[Check: $f(2,1,0) = 2$ ✓]

The normal vector is

$$\nabla f(2,1,0) = \langle e^{0}, 2e^{0}, 2e^{0} \rangle$$

$$= \langle 1, 2, 2 \rangle$$

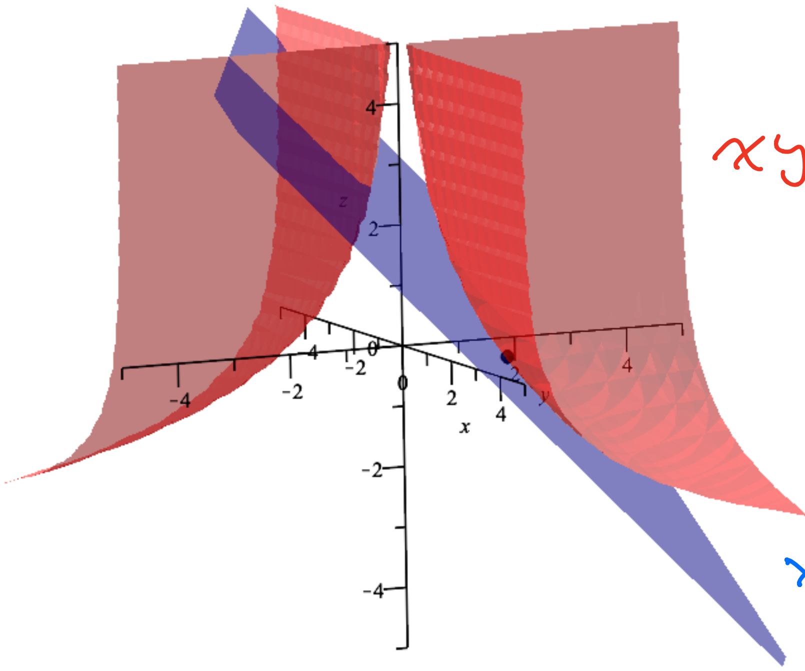
so the equation of the tangent plane is

$$1(x-2)+2(y-1)+2(z-0)=0$$

$$x-2+2y-2+2z=0$$

$$x+2y+2z=4$$

Picture:



$xye^{z}=2$

$x+2y+2z=4$

2. Consider the rectangular box with dimensions x, y, z and surface area

$$A = 2xy + 2xz + 2yz.$$

a) If x, y, z are functions of time then the chain rule gives

$$\frac{dA}{dt} = \frac{dA}{dx}\frac{dx}{dt} + \frac{dA}{dy}\frac{dy}{dt} + \frac{dA}{dz}\frac{dz}{dt}$$

First we compute

$$dA/dx = 2y + 2z + 0$$
$$dA/dy = 2x + 0 + 2z$$
$$dA/dz = 0 + 2x + 2y$$

Then we have

$$\frac{dA}{dt} = 2\left[(y+z)\frac{dx}{dt} + (x+z)\frac{dy}{dt} + (x+y)\frac{dz}{dt}\right]$$

b) If the initial conditions are

$$\langle x(0), y(0), z(0)\rangle = \langle 2, 1, 2\rangle$$
$$\langle x'(0), y'(0), z'(0)\rangle = \langle 3, -1, 1\rangle$$

then the initial rate of change of the surface area is

$$A'(0) = 2\left[(1+2)(3) + (2+2)(-1) + (2+1)(1)\right]$$

$$= 2\left[9 - 4 + 3\right]$$

$$= 16 \ \frac{(\text{unit of distance})^2}{\text{unit of time}}$$

Remark: We don't have enough information to compute the rate of change $A'(t)$ for any time other than $t = 0$.