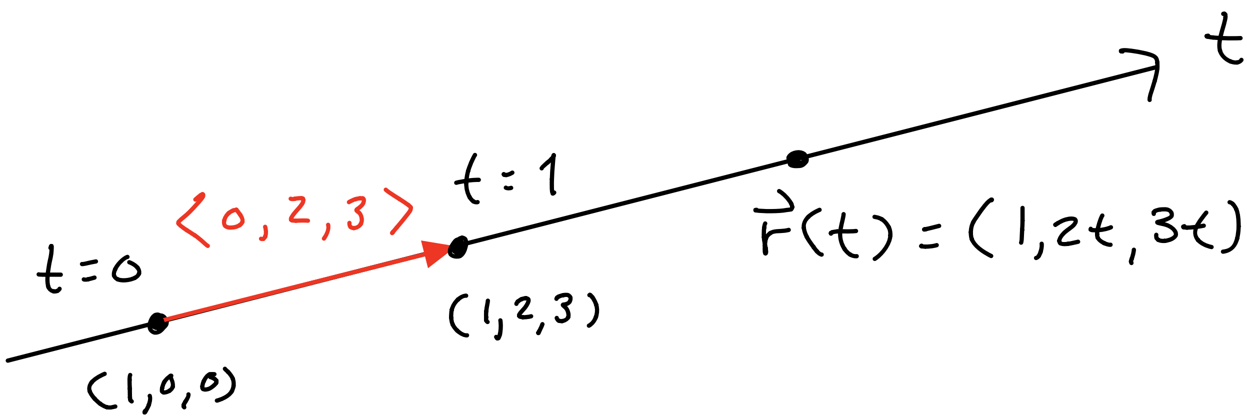
1. (a) Find a parametrization for the line passing through $P = (1,0,0)$ and $Q = (1,2,3)$.

Let $\vec{v} = \overrightarrow{PQ} = \langle 0, 2, 3 \rangle$. Then the line is

$$\vec{r}(t) = P + t\vec{v}$$
$$= \langle 1 + 0t, 0 + 2t, 0 + 3t \rangle.$$

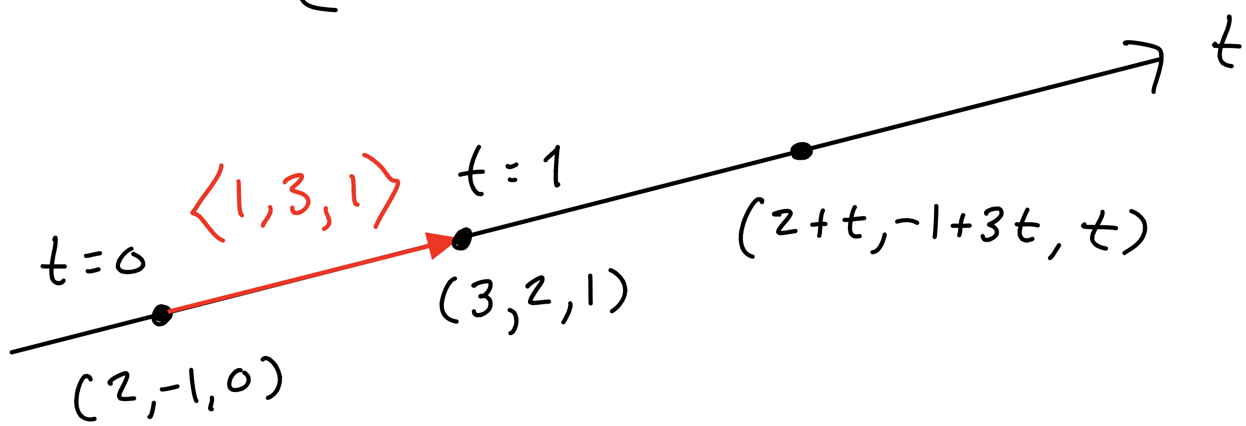


(b) Find a parametrization for the line of intersection of two planes:

$$\begin{cases} x \quad\quad - z = 2, \\ \quad -y + 3z = 1. \end{cases}$$

Let $t = z$ be the parameter. Then

$$\begin{cases} x = 2 + t \\ y = -1 + 3t \\ z = t \end{cases}$$



2. Consider the curve $\vec{r}(t) = \langle x(t), y(t) \rangle$ satisfying

$$\vec{a}(t) = \langle 0, t \rangle$$
$$\vec{v}(0) = \langle 1, 1 \rangle$$
$$\vec{r}(0) = \langle 0, 0 \rangle.$$

(a) Integrate $\vec{a}(t)$ to get $\vec{v}(t)$:

$$\vec{v}(t) = \int \vec{a}(t)\, dt$$

$$= \langle c_1, \tfrac{1}{2}t^2 + c_2 \rangle$$

for some constants c_1 & c_2. Plug in $t=0$ to get

$$\langle 1,1\rangle = \vec{v}(0) = \langle c_1, c_2\rangle,$$

so that

$$\vec{v}(t) = \langle 1, \tfrac{1}{2}t^2+1\rangle.$$

(b) Integrate $\vec{v}(t)$ to get $\vec{r}(t)$:

$$\begin{aligned}\vec{r}(t) &= \int \vec{v}(t)\,dt \\ &= \langle t+c_3, \tfrac{1}{6}t^3+t+c_4\rangle\end{aligned}$$

for some constants c_3 & c_4. Plug in $t=0$ to get

$$\langle 0,0\rangle = \vec{r}(0) = \langle c_3, c_4\rangle$$

so that

$$\vec{r}(t) = \langle t, \tfrac{1}{6}t^3+t\rangle$$

Bonus Content : Here is a picture