Math 211 **Summer 2021**
Homework 2 **Drew Armstrong**

Problem 1. A Line in $\mathbb{R}^3$. Consider the line in $\mathbb{R}^3$ passing through

$$P = (3, -1, 2) \quad \text{and} \quad Q = (1, 4, -1).$$

(a) Express this line in parametric form $\mathbf{r}(t) = (x_0 + ta, y_0 + tb, z_0 + tc)$.
(b) Find the equations of two planes in $\mathbb{R}^3$ whose intersection is this line. [Hint: Consider the "symmetric equations" $(x - x_0)/a = (y - y_0)/b = (z - z_0)/c$.]

Problem 2. Two Planes in $\mathbb{R}^3$. Consider the following two planes in $\mathbb{R}^3$:

$$\begin{cases} x - y + 0 = 1, \\ x + y + 2z = 1. \end{cases}$$

(a) Express the intersection of these planes as a parametrized line. [Hint: Let $t = z$ be the parameter and solve for x and y.]
(b) Find the angle between these two planes. [Hint: The angle between the planes is the same as the angle between the normal vectors.]

Problem 3. An Interesting Curve. Consider again the curve from Quiz 1:

$$\mathbf{r}(t) = \langle t^2 - 1, t^3 - t \rangle,$$
$$\mathbf{v}(t) = \langle 2t, 3t^2 - 1 \rangle.$$

(a) Find the points where the velocity vector is horizontal or vertical.
(b) Find the points where the velocity vector has slope $+1$ or slope -1.
(c) Sketch the curve, labeling all of these points and their velocity vectors.

Problem 4. Projectile Motion. Suppose that a projectile is launched from the point $(0, 0)$ with *initial speed* 100 feet per second and *angle θ above the horizontal*. In terms of Cartesian coordinates, the *initial velocity* is $\mathbf{v}(0) = \langle 100 \cos\theta, 100 \sin\theta \rangle$.

(a) Use this information to compute the position $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ at time t. [Hint: We have $\mathbf{r}(0) = \langle 0, 0 \rangle$ and we may assume that $\mathbf{a}(t) = \langle 0, -32 \rangle$ is constant.]
(b) When does the projectile hit the ground? Where does it land? Your answers will involve the unknown angle θ.
(c) Find the value of θ that *maximizes the horizontal distance traveled.*

Problem 5. Some Vector Identities.

(a) Given any vector $\mathbf{r}$, we can define a *unit vector* $\mathbf{u} = \mathbf{r}/\|\mathbf{r}\|$ in the same direction. Prove that $\|\mathbf{u}\| = 1$. [Hint: Use the formula $\|\mathbf{u}\|^2 = \mathbf{u} \bullet \mathbf{u} = (\mathbf{r}/\|\mathbf{r}\|) \bullet (\mathbf{r}/\|\mathbf{r}\|)$.] Find a unit vector in the direction of $\langle 1, 2, 3 \rangle$.
(b) For any vector $\mathbf{v}$ in $\mathbb{R}^3$, show that $\mathbf{v} \times \mathbf{v} = \langle 0, 0, 0 \rangle$.
(c) Suppose that a particle moves on the surface of a sphere of radius c, so that $\|\mathbf{r}(t)\| = c$ for all t. Show that the velocity is always tangent to the sphere. [Hint: Note that $\mathbf{r}(t) \bullet \mathbf{r}(t) = c^2$. Take the derivative of both sides and use the product rule.]