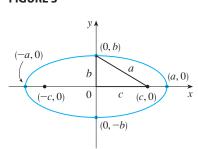


FIGURE 3





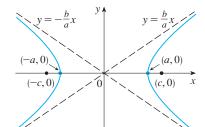
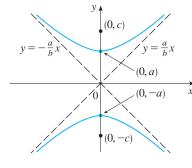


FIGURE 5









An ellipse has a simple equation if we place the foci on the x-axis at the points (-c, 0) and (c, 0) as in Figure 3 so that the origin is halfway between the foci. If the sum of the distances from a point on the ellipse to the foci is 2a, then the points (a, 0) and (-a, 0) where the ellipse meets the x-axis are called the **vertices**. The y-intercepts are  $\pm b$ , where  $b^2 = a^2 - c^2$ . (See Figure 4.)

1 The ellipse

 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad a \ge b > 0$ 

has foci  $(\pm c, 0)$ , where  $c^2 = a^2 - b^2$ , and vertices  $(\pm a, 0)$ .

If the foci of an ellipse are located on the *y*-axis at  $(0, \pm c)$ , then we can find its equation by interchanging *x* and *y* in  $\boxed{1}$ .

A hyperbola is the set of all points in a plane the difference of whose distances from two fixed points  $F_1$  and  $F_2$  (the foci) is a constant. Notice that the definition of a hyperbola is similar to that of an ellipse; the only change is that the sum of distances has become a difference of distances. If the foci are on the *x*-axis at  $(\pm c, 0)$ and the difference of distances is  $\pm 2a$ , then the equation of the hyperbola is  $(x^2/a^2) - (y^2/b^2) = 1$ , where  $b^2 = c^2 - a^2$ . The *x*-intercepts are  $\pm a$  and the points (a, 0) and (-a, 0) are the vertices of the hyperbola. There is no *y*-intercept and the hyperbola consists of two parts, called its *branches*. (See Figure 5.)

When we draw a hyperbola it is useful to first draw its **asymptotes**, which are the dashed lines y = (b/a)x and y = -(b/a)x shown in Figure 5. Both branches of the hyperbola approach the asymptotes; that is, they come arbitrarily close to the asymptotes.

2 The hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

has foci  $(\pm c, 0)$ , where  $c^2 = a^2 + b^2$ , vertices  $(\pm a, 0)$ , and asymptotes  $y = \pm (b/a)x$ .

If the foci of a hyperbola are on the y-axis, then by reversing the roles of x and y we get the graph shown in Figure 6.

We have given the standard equations of the conic sections, but any of them can be shifted by replacing x by x - h and y by y - k. For instance, an ellipse with center (h, k) has an equation of the form

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

## **CONICS IN POLAR COORDINATES**

In the following theorem we show how all three types of conic sections can be characterized in terms of a focus and directrix.

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