Intro to Linear Algebra — Summer 2017
Quiz 4 — Drew Armstrong

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**Problem 1.** Consider the following matrices:

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 0 \end{pmatrix}$$

(a) Compute the matrix products $AB$ and $BA$.

$$\begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 5 & -1 \\ 3 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 0 & 4 \end{pmatrix}$$

(b) Find a vector $\vec{x}$ such that $A\vec{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{array}\right) \downarrow \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & -1 \end{array}\right)$$

$x_1 + 0 + 2x_3 = 1$

$x_2 - x_3 = -1$

Let $x_3 = t$ (say $t = 0$)

Then $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

(c) Find a vector $\vec{y}$ such that $A\vec{y} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right) \downarrow \left(\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right)$$

$x_1 + 0 + 2x_2 = 0$

$x_2 - x_3 = 1$

Let $x_3 = t$ (say $t = 0$)

Then $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$.

**Problem 2.** Consider the same matrices again:

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 0 \end{pmatrix}$$

(a) Find a matrix $X$ such that $AX = I$. [Hint: Use Problem 1.]

From 1a & 1b we find that

$$\begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

(b) Show that the following equation has **no solution**: $B\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.

$$\begin{cases} u_1 - u_2 = 1 & (1) \\ u_2 = 0 & (2) \\ 2u_1 = 0 & (3) \end{cases}$$

Equations (2) & (3) say $u_1 = u_2 = 0$.

But then equation (1) says $0 = 1$. X

(c) Explain why there is **no matrix** $U$ such that $BU = I$. [Hint: If such a matrix $U$ existed then its first column would satisfy ... ]

If such a matrix $U$ existed, its first column $(u_1, u_2)$ would satisfy the equation of part (b)

which has NO SOLUTION