Intro to Linear Algebra
Quiz 3

Summer 2017
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Problem 1. Consider the following system of linear equations:

$$\begin{cases} x + 2y + 0 = -1 \\ x + 2y + z = 0 \\ x + 2y + 2z = 1 \end{cases}$$

(a) Put the system in reduced row echelon form (RREF).

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 1 & 2 & 1 & 0 \\ 1 & 2 & 2 & 1 \end{array}\right) \Longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 2 \end{array}\right)$$

$$\Longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad \begin{cases} x + 2y + 0 = -1 \\ 0 + 0 + z = 1 \\ 0 + 0 + 0 = 0 \end{cases}$$

(b) Use your answer from part (a) to write down the complete solution.

Let $y = t$ be free. Then

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1-2t \\ t \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}}.$$

This is the line containing the point $(-1,0,1)$ and parallel to the vector $(-2,1,0)$.

Problem 2. Consider the same system of equations again:

$$\begin{cases} x + 2y + 0 = -1 \\ x + 2y + z = 0 \\ x + 2y + 2z = 1 \end{cases}$$

1
2
3

(a) The three linear equations represent three planes living in 3D. Tell me three vectors that are perpendicular to these three planes.

Plane 1 is $\perp$ to $(1,2,0)$
plane 2 is $\perp$ to $(1,2,1)$
plane 3 is $\perp$ to $(1,2,2)$.

(b) Fill in the blanks. Let E_1, E_2, E_3 represent the three linear equations. The reason that the solution is a line (instead of a point) is because there exists a non-trivial relation among the equations:

$$E_3 = \underline{-1} \cdot E_1 + \underline{2} \cdot E_2$$

(c) Fill in the blanks. The equation from part (b) has the following consequences:

If the point (x, y, z) satisfies the first and second equations then it also

satisfies the third equation.

Geometrically, the intersection of the first and second planes is contained in

the third plane.