Math 210 E **Spring 2013**
Homework 9 **Drew Armstrong**

Reading:
None

Book Problems:
None

Additional Problems:

A.1. The *Gibonacci numbers* are defined by $G_0 = 0$, $G_1 = 1$ and

$$G_{k+2} = \frac{1}{2}G_{k+1} + \frac{1}{2}G_k \quad \text{for all } k \geq 0.$$

That is, each new term is the average of the previous two. Note that the first few Gibonacci numbers are

$$0, 1, \frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{11}{16}, \ldots$$

Write the recurrence as a discrete dynamical system (i.e. a 2×2 matrix) and express the initial condition as a combination of eigenvectors. Show that G_n approaches $2/3$ as $n \to \infty$.

A.2. *Owls vs Dusky-Footed Wood Rats!* Consider the dynamical system

$$\begin{aligned} O_{k+1} &= (.5)O_k + (.4)R_k, \\ R_{k+1} &= -p\,O_k + (1.1)R_k, \end{aligned}$$

where (O_k, R_k) gives the populations of owls and rats in the year k. In the absence of owls, the population of rats grows 10% per year. In the absence of rats, the population of owls *shrinks* by 50% per year. By eating rats the owl population grows by 40% of the rat population (more rats this year means more owls next year). The positive constant p represents the amount of rats that a typical owl eats (more owls this year means *less* rats next year).

Draw the phase portrait for this system for three different values: $p = 0.056$, $p = 0.125$, and $p = 0.2$. (You may restrict your picture to positive values of O and R.) Which value of p is best for the owls?