Math 210 **Fall 2019**
Homework 2 **Drew Armstrong**

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1. Lines in $\mathbb{R}^2$.

(a) Draw the following three parallel lines in the Cartesian plane:

$$x + 2y = -5, \quad x + 2y = 0, \quad x + 2y = 5.$$

(b) Fill in the blanks: The equation $ax+by = c$ represents a line in $\mathbb{R}^2$ that is perpendicular to the vector ______ and contains the point ______ . [There are infinitely many correct answers.]

(c) Fill in the blanks: The two lines $ax + by = c$ and $a'x + b'y = c'$ are perpendicular if and only if ______ . They are parallel if and only if ______ .

2. Two Equations in Two Unknowns. The following vector equation with two unknowns is equivalent to a system of two linear equations in two unknowns:

$$x\begin{pmatrix}-1\\2\end{pmatrix} + y\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix} \quad \Longleftrightarrow \quad \left\{\begin{array}{rcrcl} -x & + & y & = & 3,\\ 2x & + & 0y & = & 4.\end{array}\right.$$

(a) Solve the system to find x and y.

(b) Let $\mathbf{u} = (-1, 2)$ and $\mathbf{v} = (1, 0)$. Draw your solution to the vector equation, using copies of $\mathbf{u}$ and $\mathbf{v}$ to get from the origin to the point $(3, 4)$. (Strang calls this the *column picture.*)

(c) Draw your solution to the linear system as the intersection of two lines. (Strang calls this the *row picture.*)

3. Planes in $\mathbb{R}^3$.

(a) Fill in the blanks: The equation $ax + by + cz = d$ represents a plane in $\mathbb{R}^3$ that is perpendicular to the vector ______ and contains the point ______ . [There are infinitely many correct answers.]

(b) Fill in the blanks: The two lines $ax + by + cz = d$ and $a'x + b'y + c'z = d'$ are perpendicular if and only if ______ . They are parallel if and only if ______ .

(c) Fill in the blanks: The intersection of two planes in $\mathbb{R}^3$ is probably a ______ . The intersection of three planes in $\mathbb{R}^3$ is probably a ______ .

4. Three Equations in Three Unknowns. Consider the following system of two linear equations in three unknowns:

$$\left\{\begin{array}{l} x + y + 2z = 0,\\ x + 2y - z = 0.\end{array}\right.$$

(a) This system represents the intersection of two planes. Express the solution as a parametrized line $\{t\mathbf{v} : t \in \mathbb{R}\}$ for some direction vector $\mathbf{v}$. [Hint: Let $z = t$ be a free parameter.]

(b) Use your answer from part (a) to find some vector (x, y, z) that is simultaneously perpendicular to both $(1, 2, -1)$ and $(1, 1, 2)$. [There are infinitely many correct answers.]

(c) Compute the intersection of the line from part (a) with the third plane

$$x + y + z = -1.$$

(d) Finally, compute the solution of the following vector equation:

$$x\begin{pmatrix}1\\1\\1\end{pmatrix}+y\begin{pmatrix}1\\1\\2\end{pmatrix}+z\begin{pmatrix}1\\2\\-1\end{pmatrix}=\begin{pmatrix}-1\\0\\0\end{pmatrix}.$$

5. Hyperplanes in $\mathbb{R}^n$.

(a) Fill in the blanks: The equation $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$ reprents a flat ___-dimensional shape in ___-dimensional space. This shape is called a *hyperplane*.

(b) Fill in the blank: If $m \leq n$ then the intersection of m hyperplanes in n-dimensional space probably has dimension _______ .

(c) Fill in the blank: If $m \leq n$ then the solution to a system of m linear equations in n unknowns probably has _______ free parameters.

(d) Fill in the blank: If $m > n$ then a system of m linear equations in n unknowns probably has _______ .