

# Sums of Squares (FNS 195-S) Independent Study 1

Fall 2014  
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Each student will pursue an independent study and write a short paper about it (maybe 5 pages typed, or 10 pages handwritten). The due date is Monday, October 14. I may ask for a draft or an outline at some earlier date.

You should choose a topic that you find interesting and that you want to learn more about. You should seek out a few sources, including at least one actual paper book.

Here are a few suggested topics:

- The mathematics of Pythagoras. Pay special attention to (1) The Pythagorean Theorem (2) The irrationality of  $\sqrt{2}$  and (3) Pythaorean musical tuning. How did the Pytheagorean theory of musical harmony affect their other beliefs?
- A book review of Euclid's Elements. What is the structure? What does the book try to do? How does it begin; how does it end? What role did the book play in history?
- Computing the area of a surface of revolution. Investigate how Archimedes computed the surface area of a sphere. Show how Archimedes' method leads to the modern formula for the area of a surface of revolution:

$$2\pi \int y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dy.$$

- The Law of Cosines. Where does the theorem come from? Discuss a few different proofs, both ancient and modern. What is your favorite proof? How does the Law of Cosines generalize the Pythagorean Theorem?
- The Platonic Solids. There are just five regular polyhedra: tetrahedron, octahedron, cube, icosahedron, dodecahedron. Why are there only five? What do they have to do with Plato? Do they have any nice properties, like nice formulas for volume, surface area, etc.?
- The Origin of Analytic Geometry. René Descartes and Pierre de Fermat both discovered the idea of coordinate geometry around the same time, yet Descartes' contribution is more famous. What happened? What were Descartes and Fermat trying to do? What was the impact on mathematics?
- The Dot Product. Where does it come from? For that matter, where does the concept of a "vector" come from? Discuss the role of William Rowan Hamilton in this story.
- Higher dimensional balls and spheres. What is the definition of the  $n$ -dimensional ball and the  $n$ -dimensional sphere? How can we compute their volumes even though we can't see them?

You are also free to choose your own topic. I'm happy to meet and discuss choices.