Math 161 **Summer 2023**
Exam 4 Solutions **Drew Armstrong**

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1. Find the most general function $f(t)$ whose second derivative is $f''(t) = t$.

$$\begin{aligned} f''(t) &= t, \\ f'(t) &= \frac{1}{2}t^2 + C_1, \\ f(t) &= \frac{1}{2} \cdot \frac{1}{3}t^3 + C_1 t + C_2. \end{aligned}$$

2. Compute the definite integral $\int_0^9 \sqrt{x}\,dx$.

$$\begin{aligned} \int_0^9 \sqrt{x}\,dx &= \int_0^9 x^{1/2}\,dx \\ &= \left[\frac{1}{1/2+1} \cdot x^{1/2+1}\right]_0^9 \\ &= \left[\frac{1}{3/2} \cdot x^{3/2}\right]_0^9 \\ &= \left[\frac{2}{3} \cdot x^{3/2}\right]_0^9 \\ &= \frac{2}{3} \cdot (9)^{3/2} - \frac{2}{3} \cdot (0)^{3/2} \\ &= \frac{2}{3} \cdot (9^{1/2})^3 \\ &= \frac{2}{3} \cdot 27 \\ &= 18. \end{aligned}$$

3. Compute the definite integral $\int_{\pi/4}^{\pi/2} (\sin\theta + \cos\theta)\,d\theta$.

$$\begin{aligned} \int_{\pi/4}^{\pi/2} (\sin\theta + \cos\theta)\,d\theta &= [-\cos\theta + \sin\theta]_{\pi/4}^{\pi/2} \\ &= [-\cos(\pi/2) + \sin(\pi/2)] - [-\cos(\pi/4) + \sin(\pi/4)] \\ &= [-0 + 1] - \left[-1/\sqrt{2} + 1/\sqrt{2}\right] \\ &= 1. \end{aligned}$$

4. Use the Fundamental Theorem of Calculus to find the derivative $f'(x)$ of the function

$$f(x) = \int_0^{x^3} \frac{\sin t}{t}\, dt.$$

First we consider the easier function

$$g(x) = \int_0^{x} \frac{\sin t}{t}\, dt,$$

which satisfies $g'(x) = \frac{\sin x}{x}$ by the FTC. Then we use the chain rule:

$$f'(x) = [g(x^3)]' = g'(x^3) \cdot (x^3)' = \frac{\sin(x^3)}{x^3} \cdot 3x^2.$$

5. Use substitution to find the antiderivative $\int x \cdot \cos(5x^2+6)\, dx$.

We will use the substitution $u = 5x^2 + 6$ so that $du/dx = 10x$ and hence $dx = du/10x$. Then

$$\begin{aligned}\int x \cdot \cos(5x^2+6)\, dx &= \int \cancel{x} \cdot \cos(u) \cdot \frac{du}{10\cancel{x}} \\ &= \frac{1}{10} \int \cos u\, du \\ &= \frac{1}{10} \cdot \sin u + C \\ &= \frac{1}{10} \sin(5x^2+6) + C.\end{aligned}$$